

情報基礎科 212a 教理情報学

④ シェーディンガー方程式

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = \left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x,t) \right) \psi(x,t) \quad \text{①} \quad \text{②} \quad \text{③}$$

運動エネルギー 位置エネルギー E: 全エネルギー

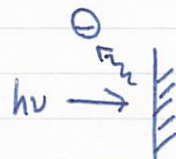
Step up 波のエネルギー?

$$E_e = \hbar \nu - W \quad (\text{光電効果})$$

$$E = \hbar \nu = \hbar \omega \quad (\omega = 2\pi \nu)$$

$$\text{①} \quad \psi(x,t) \sim e^{i(kx - \omega t)}$$

$$i\hbar \frac{\partial}{\partial t} \psi(x,t) = E \psi(x,t) \text{ だ!}$$



② $\psi(x,t)$ は波動関数



③ 明暗が波の干渉
表の波

★ 波動関数

$$P(x,t) = |\psi(x,t)|^2 \quad \text{①} \quad (x,t) \text{ は何方向でも } \psi \text{ が } \psi^* \text{ と一致}$$

④ 規格化

$$\int_{-\infty}^{\infty} dx |\psi(x,t)|^2 = 1$$

★ 定常状態 $V(x,t) \rightarrow V(x)$

$$\hat{H} \psi(x,t) = E \psi(x,t) \quad \text{①} \quad \text{②}$$

$$\Rightarrow i\hbar \frac{\partial}{\partial t} \psi(x,t) = E \psi(x,t)$$

$$\therefore \psi(x,t) = \exp(-i \frac{E}{\hbar} t) \psi(x) \quad \text{③} \quad \text{④}$$

Step up $\psi(x)$ を求める

$$\hat{H} \psi(x) = E \psi(x) \quad \text{①} \quad \text{②}$$

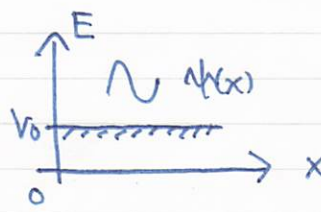
③ 設定

④ 行列の対称性

$$\hat{H} \psi = E \psi$$

⑤ $\psi(x)$ は $\psi(x)$ の対称性
固有値があるか?

⑬④ $V(x) = V_0$ (定数)



$$\hat{H}\psi(x) = E\psi(x)$$

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_0\right)\psi(x) = E\psi(x)$$

$$\therefore \frac{\partial^2}{\partial x^2}\psi(x) = -\frac{2m(E-V_0)}{\hbar^2}\psi(x)$$

$$\text{よ2. } \psi(x) = A\exp(ikx) + B\exp(-ikx) \quad \text{① 一般解!}$$

$$k = \frac{\sqrt{2m(E-V_0)}}{\hbar} \quad \text{エネルギー} k$$

Step up 運動量 = 運動エネルギー

$$p = \hbar k \quad (= -i\hbar \frac{\partial}{\partial x})$$

$$\therefore k = \frac{p}{\hbar} = \frac{\hbar k^2}{2m}$$

$$\text{724} \quad \frac{\hbar^2 k^2}{2m} = E - V_0$$

$$E = \frac{\hbar^2 k^2}{2m} + V_0 \quad \text{① } k \text{ は } E \text{ と } V_0 \text{ によって決まる}$$

$$\psi(x) = A\exp(ikx) + B\exp(-ikx)$$

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$$\frac{h}{\lambda} \rightarrow \frac{h}{\lambda} = p$$

① $\lambda \rightarrow \lambda'$ 色が変わる!

波数

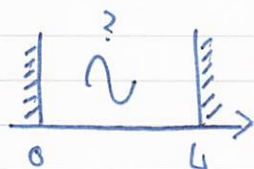
$$p = \frac{h}{\lambda} = \hbar k \quad (k = \frac{2\pi}{\lambda})$$

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⑬⑤ $V(x) = \begin{cases} 0 & (0 \leq x \leq L) \\ \infty & (x < 0, L < x) \end{cases} \Rightarrow \psi(x) = 0 \quad (\text{境界条件})$



(0 ≤ x ≤ L)

$$\hat{H}\psi(x) = E\psi(x)$$

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$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\psi(x) = E\psi(x)$$

$$\text{同様に } \frac{\partial^2}{\partial x^2}\psi(x) = -\frac{2mE}{\hbar^2}\psi(x)$$

$$\text{よ2 } \psi(x) = A\exp(ikx) + B\exp(-ikx) \quad \text{① 一般解!}$$

$$k = \frac{\sqrt{2mE}}{\hbar} \quad \text{エネルギー} k$$

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$$E = \frac{\hbar^2 k^2}{2m} \quad \text{① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪ ⑫ ⑬ ⑭ ⑮ ⑯ ⑰ ⑱ ⑲ ⑳ ㉑ ㉒ ㉓ ㉔ ㉕ ㉖ ㉗ ㉘ ㉙ ㉚ ㉛ ㉜ ㉝ ㉞ ㉟ ㊱ ㊲ ㊳ ㊴ ㊵ ㊶ ㊷ ㊸ ㊹ ㊺ ㊻ ㊼ ㊽ ㊾ ㊿}$$

Step up 7.2.7 a I 粒子-量子

$$1 + e^{-\hbar\omega/kT} + e^{-2\hbar\omega/kT} \quad \text{黑体辐射}$$

$$= \frac{1}{1 - e^{-\hbar\omega/kT}} \propto U(\omega, T)$$



① I 粒子 $E = n\hbar\omega$ 何?
 量子 = zero energy?

$$\psi(0) = \psi(L) = 0 \quad \text{H (2次元)}$$

$$\psi(0) = A + B = 0 \quad \therefore B = -A$$

$$\psi(L) = Ae^{ikL} + Be^{-ikL}$$

$$= 2A \sin kL = 0$$

① $A \neq 0, L \neq 0$ 故 $kL = n\pi$

$$\text{故 } kL = 0, \pm\pi, \pm2\pi, \dots, n\pi \quad (n = 0, \pm1, \pm2, \dots) \quad \text{H 2次元 E 何?}$$

$$\frac{\sqrt{2mE}}{\hbar} L = n\pi \quad \left(\frac{\sqrt{2mE}}{\hbar} = \frac{n}{2L} \right) \quad \therefore E_n = \frac{n^2 \hbar^2}{8mL^2} \quad \text{① I 粒子-量子化!}$$

$$\text{故 } \psi_n(x) \propto \sin k_n x \quad k_n = \frac{\pi}{L} n \quad \text{何?}$$

$$\text{規格化 } \psi_n(x) = A \sin k_n x$$

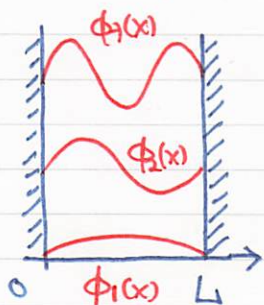
$$\int_0^L |\psi_n(x)|^2 dx = |A|^2 \int_0^L \sin^2 k_n x dx = |A|^2 \left[\frac{x}{2} - \frac{1}{4k_n} \sin 2k_n x \right]_0^L = \frac{L}{2} |A|^2$$

$$\psi(x) = \sqrt{\frac{2}{L}} \psi_n(x) \quad \text{① } n \text{ 何 } E_n = \frac{n^2 \hbar^2}{8mL^2} \quad \text{② } A = \sqrt{\frac{2}{L}}$$

Step up 量子 = 波

$$\psi(0) = \psi(L) = 0 \quad k_n (\Rightarrow E_n) \text{ 量子化}$$

★ I 粒子-準位



$$E_3 = \frac{9\hbar^2}{8mL^2}$$

$$E_2 = \frac{4\hbar^2}{8mL^2}$$

$$E_1 = \frac{\hbar^2}{8mL^2} \quad (\text{基底状態})$$

而起状態

$$E_m - E_n = \hbar\omega_{mn} \text{ の光子放出!}$$

$$\hbar\omega_{mn} \text{ の光子吸収!}$$

$$E_m \rightarrow E_n \quad \rightarrow \hbar\omega_{mn} \quad \text{固有エネルギー}$$

① ϕ_1, ϕ_2 の I 粒子-状態を 2 つ使う
 $\Rightarrow$ 量子ビット!

⑬① $V(x) = 0$ 但し同期条件
($0 \leq x \leq 2\pi a$)



長さ a

$$\hat{H}\psi(x) = E\psi(x)$$

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}\right) \psi(x) = E\psi(x)$$

同様に $\frac{\partial^2}{\partial x^2} \psi(x) = -\frac{2mE}{\hbar^2} \psi(x)$

よって $\psi(x) = A\exp(ikx) + B\exp(-ikx)$ ① 一般解

$k = \frac{\sqrt{2mE}}{\hbar}$ ① 非連続なエネルギー

② 同期境界

$$\psi(x) = \psi(x+2\pi a)$$

$$e^{ik2\pi a} = 1 \quad \therefore 2\pi ka = 2\pi n \quad (n=1,2,\dots)$$

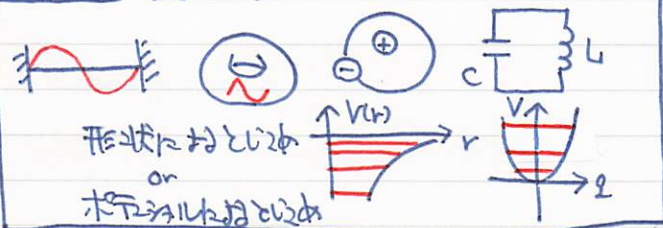
$$\text{よって } k = \frac{n}{a}$$

$$\frac{\sqrt{2mE}}{\hbar} = \frac{n}{a} \quad \therefore E = \frac{n^2 \hbar^2}{2ma^2}$$

① エネルギーの量子化!

② 粒子が波動関数を持つ

Step up | “量子”力学



① “波” + “粒子”
二つの性質がある

② 量子は運動形態を保持
“状態”を持つ

波動関数 $\psi(x)$?

7-11 変換 = 量子化

$$\hat{H}\psi(x) = E\psi(x)$$

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi(x) = E\psi(x) \quad \textcircled{11} \text{ 偏微分方程式は 7-11! }$$

Step up 7-11 変換

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(k) e^{ikx} dk$$

$$\hat{f}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

★ 何故 7-11?

$$\frac{\partial}{\partial x} f(x)$$

$$\textcircled{2} \rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\partial}{\partial x} f(x) e^{-ikx} dx = \frac{1}{2\pi} \left[\underbrace{f(x) e^{-ikx}}_{f(\pm\infty) \rightarrow 0} \right]_{-\infty}^{\infty} + (ik) \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx}_{\hat{f}(k)}$$

$$\uparrow \quad \downarrow \quad \textcircled{11} \quad \frac{\partial}{\partial x} \rightarrow ik \text{ 相当.}$$

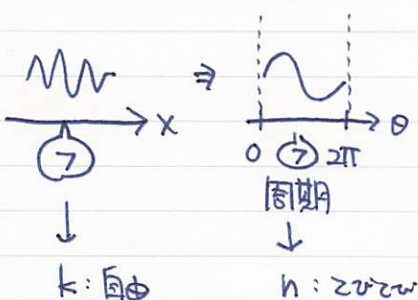
Step up 7-11 と 量子化

$$p = \hbar k \rightarrow -i\hbar \frac{\partial}{\partial x} \text{ (量子化)}$$

$$ik \leftarrow \frac{\partial}{\partial x} \text{ (7-11)}$$

① (古典) x, p は同時決まる
(量子) x と p は別決まる

★ 7-11 変換の展開は...



Step up 7-11 変換の展開

$$f(x) = \sum_{n=-\infty}^{\infty} a_n e^{in\theta} \quad \text{for } 0 \leq \theta < 2\pi$$

$$a_n = \frac{1}{2\pi} \int_0^{2\pi} f(\theta) e^{-in\theta} d\theta$$

$$f(x) = \sum_{n=-\infty}^{\infty} a_n \exp(i \frac{2\pi}{\lambda} nx) \quad \text{for } 0 \leq x < \lambda$$

$$a_n = \frac{1}{\lambda} \int_0^{\lambda} f(x) \exp(-i \frac{2\pi}{\lambda} nx) dx$$

① zuzw 万端 zuzw zuzw

7-11 変換の仕方.

① 変換の仕方... 行方!

$$\begin{cases} \psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp \psi(p) e^{i\frac{p}{\hbar}x} \\ \psi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dx \psi(x) e^{-i\frac{p}{\hbar}x} \end{cases}$$

$$\left[\begin{array}{l} p = \hbar k \\ \psi(k)/\hbar \equiv \psi(p) \end{array} \right]$$

① $\psi(x), \psi(p)$ は 2つの関数で表す
内積 $\langle \psi | \psi \rangle$?

② 同じ関数で表す = 代数!

③ 7 は 簡単!

7-12 記法

$$\begin{cases} \langle x | \psi \rangle = \int_{-\infty}^{\infty} dp \langle x | p \rangle \langle p | \psi \rangle \\ \langle p | \psi \rangle = \int_{-\infty}^{\infty} dx \langle p | x \rangle \langle x | \psi \rangle \end{cases}$$

I: 恒等演算

Step up 7-12 記法

$$\psi(x) = \langle x | \psi \rangle \quad (\text{位置表示})$$

$$\psi(p) = \langle p | \psi \rangle \quad (\text{運動量表示})$$

Step up 恒等演算

$$\int_{-\infty}^{\infty} dx |x\rangle \langle x| = I$$

$$\int_{-\infty}^{\infty} dp |p\rangle \langle p| = I$$

これに λ を代入して共役

$$\langle x | p \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{i\frac{p}{\hbar}x}$$

$$\langle p | x \rangle = \frac{1}{\sqrt{2\pi\hbar}} e^{-i\frac{p}{\hbar}x}$$

★ 演算子

$$-i\hbar \frac{\partial}{\partial x} \langle x | p \rangle = p \langle x | p \rangle$$

① $\langle x | p \rangle$ が固有...?

$$\hat{p} \langle x | p \rangle = p \langle x | p \rangle$$

$$\hat{p} |p\rangle = p |p\rangle$$

$$\langle x | \hat{p} |p\rangle = p \langle x | p \rangle$$

$$-i\hbar \frac{\partial}{\partial x} \langle x | p \rangle = p \langle x | p \rangle$$

X は 位置 (x)

$$\hat{x} |x\rangle = x |x\rangle \quad \text{これ}$$

$$\langle p | \hat{x} |x\rangle = x \langle p | x \rangle$$

$$i\hbar \frac{\partial}{\partial p} \langle p | x \rangle = x \langle p | x \rangle$$

① 運動量 p による状態 (Z 成分) $\psi(p) = |p\rangle$

② $\hat{p} \rightarrow -i\hbar \frac{\partial}{\partial x}$ を示す?

p は X の Z 成分

③ $|x\rangle$ は位置 x による状態 (Z 成分) $\psi(x) = |x\rangle$

Step up $\hat{x}$ 表示, $\hat{p}$ 表示

$$\langle x | \hat{p} = -i\hbar \frac{\partial}{\partial x} \langle x |$$

$$[\hat{p}^\dagger | x \rangle = i\hbar \frac{\partial}{\partial x} | x \rangle]$$

$$\langle p | \hat{x} = i\hbar \frac{\partial}{\partial p} \langle p |$$

$$[\hat{x}^\dagger | p \rangle = -i\hbar \frac{\partial}{\partial p} | p \rangle]$$

Step up 共役 + 転置 = 随伴

$$\hat{x} | x \rangle = x | x \rangle$$

↓ + : 随伴 (随伴)

$$\langle x | \hat{x}^\dagger = \langle x | x^* = \langle x | x$$

$$\begin{aligned} \textcircled{E} \langle x | \hat{x} | x \rangle &= x \langle x | x \rangle \\ \textcircled{E} \langle x | \hat{x}^\dagger | x \rangle &= x \langle x | x \rangle \end{aligned}$$

Step up 物理量 (実数) は Hermitian

$$\hat{x} = \hat{x}^\dagger \quad \text{同様} \quad \hat{p} = \hat{p}^\dagger$$

★ 2次元方程式も...

$$\hat{H} | \psi \rangle = E | \psi \rangle$$

$$\Rightarrow \hat{H} | \psi \rangle = E | \psi \rangle \quad \textcircled{II} \text{ 何で表が自由}$$

$$\langle x | \hat{H} | \psi \rangle = E \langle x | \psi \rangle$$

$$\langle x | \left(\frac{\hat{p}^2}{2m} + V(\hat{x}) \right) | \psi \rangle$$

↓ 第1量化 [$\langle x |$ を受ける]

$$\left(-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right) \langle x | \psi \rangle = E \langle x | \psi \rangle \quad (\text{波動方程式})$$

$$\textcircled{II} \text{ } \psi(x) \text{ は波動関数 } | \psi \rangle \text{ を表す}$$

微分方程式

線型代数

★ 交換関係 (行列表示)

Step up 交換関係

$$[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a}$$

↑ $\hat{a} \geq \hat{b}$ の同時固有状態がある
 $\hat{a} \rightarrow a, \hat{b} \rightarrow b$ となる

(例) $[\hat{x}, \hat{p}] = i\hbar$ (1) 何かで検証可能

$$\langle \psi | \hat{x} \hat{p} | \psi \rangle = x \langle \psi | \hat{p} | \psi \rangle = x (-i\hbar \frac{\partial}{\partial x}) \langle \psi | \psi \rangle$$

$$\langle \psi | \hat{p} \hat{x} | \psi \rangle = (-i\hbar \frac{\partial}{\partial x}) \langle \psi | \hat{x} | \psi \rangle = (-i\hbar \frac{\partial}{\partial x}) x \langle \psi | \psi \rangle$$

$$-i\hbar \langle \psi | \psi \rangle = -i\hbar \langle \psi | \psi \rangle + x (-i\hbar \frac{\partial}{\partial x}) \langle \psi | \psi \rangle$$

$$\langle \psi | [\hat{x}, \hat{p}] | \psi \rangle = i\hbar \langle \psi | \psi \rangle \quad (1) \quad \langle 1 \rangle \text{ 以外は 0}$$

Step up 反交換関係

$$\{\hat{a}, \hat{b}\} = \hat{a}\hat{b} + \hat{b}\hat{a}$$

$$[\hat{a}, \hat{b}]^\dagger = (\hat{a}\hat{b})^\dagger - (\hat{b}\hat{a})^\dagger \stackrel{\text{エルミート}}{=} \hat{b}^\dagger \hat{a}^\dagger - \hat{a}^\dagger \hat{b}^\dagger = -[\hat{a}^\dagger, \hat{b}^\dagger] \quad (1) \quad \text{純虚数!}$$

$$[\hat{a}, \hat{b}] = i0 \quad \{\hat{a}, \hat{b}\} = \Delta \text{ (実数!)}$$

★ 期待値

$$P(x) = |\psi(x)|^2 = \langle \psi | x \rangle \langle x | \psi \rangle$$

$$\int dx P(x) = \int dx \langle \psi | \underline{x} \rangle \langle \underline{x} | \psi \rangle \stackrel{I}{=} 1 \quad \therefore \langle \psi | \psi \rangle = 1 \quad (1) \quad \psi \text{ は規格化された波関数!}$$

$$\circ \langle f(x) \rangle = \int dx f(x) P(x) \quad (\text{期待値})$$

$$= \int dx f(x) \langle \psi | x \rangle \langle x | \psi \rangle \stackrel{I}{=} \int dx \langle \psi | f(\hat{x}) | x \rangle \langle x | \psi \rangle$$

$$= \langle \psi | f(\hat{x}) | \psi \rangle \quad (1) \quad \text{ハミルトニアンに代入して}$$

Step up 期待値

$$\langle f(x) \rangle = \langle \psi | f(\hat{x}) | \psi \rangle$$

$$\text{つまり } \langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle \text{ だよ!}$$

Ⅳ 不確定性関係

Step up コーシ・シュミットの不等式

$$\|\vec{a}\|^2 \|\vec{b}\|^2 \geq (\vec{a} \cdot \vec{b})^2$$

① 類似度は1から
 $\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \cos \theta$

$$\Rightarrow \langle \varphi | \varphi \rangle \langle \phi | \phi \rangle \geq \langle \varphi | \phi \rangle \langle \phi | \varphi \rangle$$

$$\begin{cases} |\varphi\rangle = (\hat{A} - \langle \hat{A} \rangle) |\psi\rangle \\ |\phi\rangle = (\hat{B} - \langle \hat{B} \rangle) |\psi\rangle \end{cases} \quad \text{① 実現値と期待値の差}$$

$$\langle \Delta A^2 \rangle \langle \Delta B^2 \rangle \geq \langle (\hat{A} - \langle \hat{A} \rangle) (\hat{B} - \langle \hat{B} \rangle) \rangle \langle (\hat{B} - \langle \hat{B} \rangle) (\hat{A} - \langle \hat{A} \rangle) \rangle$$

$$\begin{cases} \star = \langle \hat{A} \hat{B} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle = \frac{1}{2} (\langle \{\hat{A}, \hat{B}\} \rangle + \langle [\hat{A}, \hat{B}] \rangle) - \langle \hat{A} \rangle \langle \hat{B} \rangle = z \\ \star' = \langle \hat{B} \hat{A} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle = \frac{1}{2} (\langle \{\hat{A}, \hat{B}\} \rangle - \langle [\hat{A}, \hat{B}] \rangle) - \langle \hat{A} \rangle \langle \hat{B} \rangle = z^* \end{cases}$$

$$\begin{aligned} \star \star' &= |z|^2 = \left(\frac{1}{2} \langle \{\hat{A}, \hat{B}\} \rangle - \langle \hat{A} \rangle \langle \hat{B} \rangle \right)^2 + \left(\frac{1}{2} \langle [\hat{A}, \hat{B}] \rangle \right)^2 \\ &\geq \left| \frac{1}{2} \langle [\hat{A}, \hat{B}] \rangle \right|^2 \end{aligned}$$

Step up ハイゼンベルグの不等式

$$\langle \Delta A^2 \rangle \langle \Delta B^2 \rangle \geq \frac{1}{4} |\langle [\hat{A}, \hat{B}] \rangle|^2$$

① $\langle \Delta x^2 \rangle \langle \Delta p^2 \rangle \geq \frac{\hbar^2}{4}$ (ハイゼンベルグ 1927)

同じ $|\psi\rangle$ を用いる

$$\left\{ \begin{array}{l} \langle \Delta x^2 \rangle \geq 50 \text{ eV} \text{ のオーダー} \\ \langle \Delta p^2 \rangle \geq 50 \text{ eV} \text{ のオーダー} \end{array} \right\} \quad \langle \Delta x^2 \rangle \langle \Delta p^2 \rangle \geq \frac{\hbar^2}{4}$$

① $|\psi\rangle$ 自体は
不確定性!
② 測定はあくまで
観測値の平均

(ハイゼンベルグ 1927)

④ 量子ビットの作りか.

★ $[\hat{x}, \hat{p}] = i\hbar$ 对易

$[x, p] = i\hbar$

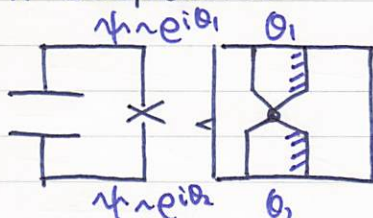
$\left[\text{Spring} \right] \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E \Rightarrow \frac{\overbrace{p=mv}^{\text{p}_2}}{2m} + \frac{1}{2}kx^2 = E \quad (\hat{H}|\psi\rangle = E|\psi\rangle)$

!! 他にもワザあり!

 $E_n = \hbar\omega\left(n + \frac{1}{2}\right)$

$$\Rightarrow \frac{1}{2} L \dot{\phi}^2 + \frac{q^2}{2C} = E \quad \Rightarrow \quad \frac{\hat{L}}{2I} + \frac{\hat{Q}^2}{2C} = \hat{H} \quad (1) \quad \begin{cases} [\hat{Q}, \hat{\phi}] = i\hbar \\ [\hat{L}, \hat{\phi}] = 0 \end{cases}$$

★ Josephson 接合



BEC 2109 12/12

超伝導で超流体とは？

"Josephson ~~to~~"

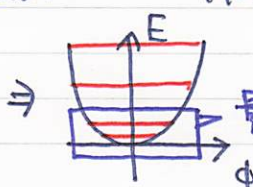
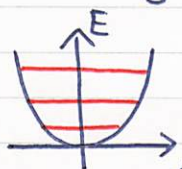
$$I = I_0 \sin \theta = I_0 \sin 2\pi \left(\frac{\phi}{\phi_0} \right) \quad [\theta = \theta_1 - \theta_2]$$

$$V = + \frac{d\phi}{dt} = L \frac{dI}{dt} - \lambda \frac{dI}{dt} = \frac{2\pi I_0}{\phi_0} \cos 2\pi \left(\frac{\phi}{\phi_0} \right) \frac{d\phi}{dt}$$

$$\text{对于 } L = \frac{d\phi/dt}{dI/dt} = \frac{\frac{\Phi_0}{2\pi I_0}}{\cos 2\pi(\phi/\phi_0)} \quad \text{非恒定性}$$

$$E_{\text{ext}} = \int_0^i L i \frac{di}{dt} dt = \frac{1}{2} L i^2 \quad \text{J}$$

$$= \int \left(\frac{d\phi}{dt} \right) \cdot I_0 \sin 2\pi \left(\frac{\phi}{\phi_0} \right) dt = \frac{\phi_0}{2\pi} \left(1 - \cos 2\pi \left(\frac{\phi}{\phi_0} \right) \right) \approx \frac{\phi_0}{2\pi} \left(\frac{1}{2} \left(\frac{2\pi\phi}{\phi_0} \right)^2 - \frac{1}{4!} \left(\frac{2\pi\phi}{\phi_0} \right)^4 \right)$$



문자화!

① 低利率-节约利用



Q.1 ϕ_0 作爲電流 I 的 $1/2$

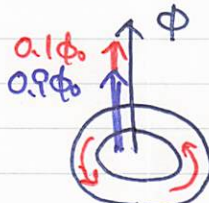


☆ 磁束の量子化



「超伝導427」

$$\boxed{\phi = n\phi_0}$$



$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\phi\rangle + |\phi\rangle)$$

flux type qubit

固有状態

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp e^{i\frac{p}{\hbar}x} \psi(p)$$

$\langle x|\psi\rangle$ $\langle x|p\rangle$ $\langle p|\psi\rangle$

$$|\psi\rangle = \int_{-\infty}^{\infty} dp |p\rangle \langle p|\psi\rangle \quad \left[= \int_{-\infty}^{\infty} dp C_p |p\rangle \right] \quad \text{!! } |p\rangle \text{ 展開!}$$

$$= \sum_{n=0}^{\infty} |u_n\rangle \langle u_n|\psi\rangle \quad \left[\sum_{n=0}^{\infty} C_n |u_n\rangle \right] \quad \text{!! } |u_n\rangle \text{ 展開!}$$

Step up 固有状態展開

$\hat{H}|u_n\rangle = E_n|u_n\rangle$ $\langle u_m|u_n\rangle = \delta_{mn}$ (正規化) (直交)

$|\psi\rangle = \sum_n C_n |u_n\rangle$ $\sum_n |u_n\rangle \langle u_n| = I$ (完全性)

$$\therefore \langle \psi|\psi\rangle = \left(\sum_n C_n \langle u_n| \right)^{\dagger} \sum_n C_n |u_n\rangle$$

$$= \sum_n \sum_m C_m^* C_n \underbrace{\langle u_m|u_n\rangle}_{\delta_{mn}} = \sum_n |C_n|^2$$

!! $\langle \psi|\psi\rangle = |\alpha|^2$
 $|C_n|^2 \geq |u_n\rangle$ が必要!

!! $|u_n\rangle \geq E_n$ が必要!
 (正規化)

★ Schrödinger-方程式

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

$$|\psi(t)\rangle = \sum_n C_n(t) |u_n\rangle$$

$$\therefore i\hbar \sum_n \frac{\partial}{\partial t} C_n(t) |u_n\rangle = \sum_n C_n(t) E_n(t) |u_n\rangle$$

左の $\langle u_m|$

$$i\hbar \frac{\partial}{\partial t} C_m(t) = E_m(t) C_m(t) \quad \text{よって } C_m(t) = \exp(-i\frac{E_m}{\hbar}t) C_m(0)$$

$$\therefore |\psi(t)\rangle = \sum_n C_n \exp(-i\frac{E_n}{\hbar}t) |u_n\rangle \quad \text{!! 固有状態毎振動}$$

量子力学

★ Pauli 行列

$$|n\rangle = a|0\rangle + b|1\rangle$$

⑪ 基底が 2 次元空間

$$\hat{H} = \begin{pmatrix} H_{00} & H_{01} \\ H_{10} & H_{11} \end{pmatrix}$$

$$\hat{H} = \hat{H}^\dagger \quad \left\{ \begin{array}{l} H_{00} = H_{00}^* \quad H_{01} = H_{10}^* \\ H_{10} = H_{01} \quad H_{11} = H_{11}^* \end{array} \right. \quad \begin{array}{l} H_{10} = a+ib \\ H_{11} = a-ib \end{array}$$

$c+d$ $-c+d$ 実数

$$\hat{H} = \begin{pmatrix} c+d & a-ib \\ a+ib & -c+d \end{pmatrix} = a\hat{\sigma}_x + b\hat{\sigma}_y + c\hat{\sigma}_z + d\hat{I} \quad \text{⑪ 4 次元行列}$$

Step up Pauli 行列

$$\hat{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\hat{\sigma}_x, \hat{\sigma}_y] = \hat{\sigma}_x \hat{\sigma}_y - \hat{\sigma}_y \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} - \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = 2i\hat{\sigma}_z$$

Step up 交換・反交換関係

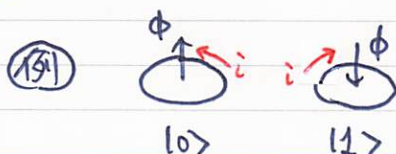
$$[\hat{\sigma}_1, \hat{\sigma}_2] = 2i\hat{\sigma}_3 \quad \{\hat{\sigma}_1, \hat{\sigma}_2\} = 0$$

★ 重合成の基底

$$\hat{H} = -\hbar\hat{\sigma}_z - \Gamma\hat{\sigma}_x \quad \Gamma=0$$

縦磁場 横磁場

$$\begin{cases} \hat{H}|0\rangle = -\hbar|0\rangle & (|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}) \\ \hat{H}|1\rangle = +\hbar|1\rangle \end{cases}$$



⑪ 基底が 2 次元空間

⑪ 基底が 2 次元空間

$$\hbar=0$$

$$\hat{H} = \begin{pmatrix} 0 & -P \\ -P & 0 \end{pmatrix} \quad \hat{H}|\psi\rangle = E|\psi\rangle \quad \det(E\hat{I} - \hat{H}) = \begin{vmatrix} E & P \\ P & E \end{vmatrix} = E^2 - P^2 = 0$$

$$\therefore E = \pm P$$

$$\hat{H}|\psi\rangle = \pm P|\psi\rangle \quad |\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} 0 & -P \\ -P & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \pm P \begin{pmatrix} a \\ b \end{pmatrix} \quad \begin{cases} -Pb = \pm Pa & a-b=1 \text{ or } a=b \\ -Pa = \pm Pb & a=b=-1 \end{cases}$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle \quad \text{重疊状態}$$

$$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle$$

$$\text{①} \quad \begin{array}{c} \uparrow \downarrow \\ |+\rangle \end{array} \quad \begin{array}{c} \uparrow \downarrow \\ |-\rangle \end{array}$$

$$\uparrow \quad \text{一方が上は下}$$

$$\uparrow \quad \text{↑と↓は}|+\rangle, |0\rangle, |1\rangle \text{と}|-\rangle$$

② 2つの模型

$$\hat{H} \Rightarrow \frac{1}{2} M_{12} \hat{I}_1 \hat{I}_2 \quad | \pm \pm \rangle \quad \text{相互作用!}$$

$$\text{①} \quad \text{2a 2b 量対称性. 空間は4つ}$$

$$00, 01, 10, 11$$

$$|\psi_1\rangle \otimes |\psi_2\rangle = \begin{pmatrix} a & c \\ b & d \end{pmatrix} = ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle$$

$$\hat{H} = -\hbar \hat{\sigma}_1^z - \hbar \hat{\sigma}_2^z - P \hat{\sigma}_1^x - P \hat{\sigma}_2^x - J \hat{\sigma}_1^z \hat{\sigma}_2^z$$

$\hat{\sigma}_1^z \otimes \hat{I}_2 \quad \hat{I}_1 \otimes \hat{\sigma}_2^z \quad \text{相互作用}$

$$\text{①} \quad \hat{\sigma}_1^z \otimes \hat{I}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{実際}$$

$$\begin{cases} \hat{\sigma}_1^z |01\rangle = |01\rangle \\ \hat{\sigma}_1^z |10\rangle = -|10\rangle \end{cases}$$

$$\hat{I}_1 \otimes \hat{\sigma}_2^z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{cases} \hat{\sigma}_2^z |01\rangle = -|01\rangle \\ \hat{\sigma}_2^z |10\rangle = |10\rangle \end{cases}$$

$$\hat{\sigma}_1 \otimes \hat{\sigma}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{cases} \hat{\sigma}_1 \hat{\sigma}_2 |01\rangle = -|01\rangle \\ \hat{\sigma}_1 \hat{\sigma}_2 |11\rangle = -|11\rangle \end{cases}$$

多体

$$\Rightarrow \hat{H} = - \underbrace{\sum_{i=1}^N h_i \hat{\sigma}_i^z}_{\text{Ising 模型}} - \underbrace{\sum_{\langle ij \rangle} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z}_{\text{相互作用}} - \Gamma \sum_{i=1}^N \hat{\sigma}_i^x$$
(1) $\sum_{\langle ij \rangle}$ は結合の和

★ 古典スピン系 (対角成分のみ)

$\Gamma \rightarrow 0$

$$H = - \sum_{i=1}^N h_i \sigma_i - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \pm 1$

$\hat{\sigma}_i^z \rightarrow \sigma_i^z$

(1) $|0\rangle, |1\rangle \Sigma$
 $\sigma_i^z = \pm 1$ の区別

Landau-Zener (J. Phys. B: At. Mol. Opt. Phys. 56 (2023) 104001)

$$\hat{H} = -\hbar\alpha t \hat{\sigma}_z - \hbar g \hat{\sigma}_x = \hbar \begin{pmatrix} -\alpha t & -g \\ -g & \alpha t \end{pmatrix}$$

$$[t \rightarrow -\infty] \hat{H} \sim \hbar \begin{pmatrix} +\alpha t & 0 \\ 0 & -\alpha t \end{pmatrix} \rightarrow \begin{matrix} \uparrow \\ 100\% \\ \bullet \end{matrix} \begin{matrix} |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{matrix}$$

$$[t \rightarrow +\infty] \hat{H} \sim \hbar \begin{pmatrix} -\alpha t & 0 \\ 0 & \alpha t \end{pmatrix} \rightarrow \begin{matrix} \uparrow \\ ? \\ \downarrow \end{matrix} \begin{matrix} |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{matrix}$$

①① $|1\rangle \rightarrow |0\rangle$
基底状態が変化する

①① 途中は? 本質は? $\frac{1}{\sqrt{2}}$

$$\begin{pmatrix} 0 & g \\ g & 0 \end{pmatrix} \Rightarrow (|0\rangle + |1\rangle) / \sqrt{2}$$

★ $g=0$

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \exp\left[\int_{-\infty}^t i\alpha t' dt'\right]$$

$$\begin{cases} \frac{\partial}{\partial t} a(t) = i\alpha t a(t) & \therefore a(t) = \exp\left(i\frac{\alpha t^2}{2}\right) a(-\infty) \\ \frac{\partial}{\partial t} b(t) = -i\alpha t b(t) & \therefore b(t) = \exp\left(-i\frac{\alpha t^2}{2}\right) b(-\infty) \end{cases}$$

①① 位相が激に変化

①① $\hbar^2$ 考慮

①① 遷移なし

★ $g \neq 0$

$$\frac{\partial}{\partial t} a(t) = i\alpha t a(t) + ig b(t) \quad a(t) = \exp\left(i\frac{\alpha t^2}{2}\right) c(t) \quad \text{定数変換}$$

$$\Rightarrow \exp\left(i\frac{\alpha t^2}{2}\right) \frac{\partial}{\partial t} a(t) = ig b(t) \Rightarrow \frac{\partial c}{\partial t} = ig \exp(-i\alpha t^2) d(t)$$

$$\frac{\partial}{\partial t} b(t) = ig a(t) - i\alpha t b(t) \quad b(t) = \exp(-i\frac{\alpha t^2}{2}) d(t) \quad \text{変換}$$

$$\Rightarrow \exp\left(i\frac{\alpha t^2}{2}\right) \frac{\partial}{\partial t} d(t) = ig a(t) \Rightarrow \frac{\partial d}{\partial t} = ig \exp(i\alpha t^2) c(t)$$

$$\therefore \frac{\partial}{\partial t} c(t) = -g^2 \exp(-i\alpha t^2) \int_{-\infty}^t \exp(i\alpha t'^2) c(t') dt'$$

Step up 21177151X

$$\begin{aligned} \int_{-\infty}^t a(t') b(t') dt' &\rightarrow \int_{-\infty}^t \underbrace{a(t') b(t')}_{\text{for } a(t)} dt' \\ &\approx -g^2 \exp(-i\alpha t^2) \int_{-\infty}^t \exp(i\alpha t'^2) dt' c(t) \end{aligned}$$

$$\text{b2 } c(t) = \exp\left(-g^2 \int_{-\infty}^t \exp(-i\alpha t'^2) f(t') dt'\right) c(-\infty) \rightarrow 0$$

$$\text{同様 } d(t) = \exp\left(-g^2 \int_{-\infty}^t \exp(+i\alpha t'^2) f(-t') dt'\right) d(-\infty) \rightarrow 1$$

★ $d(t \rightarrow \infty) = ?$

$$d(t \rightarrow \infty) = \exp(-g^2 \int_{-\infty}^{\infty} \exp(i\alpha t_1^2) \int_{-\infty}^{t_1} \exp(-i\alpha t_2^2) dt_1 dt_2)$$

$$\int_0^{t_1} + \int_{-\infty}^0$$

$$\int_{-\infty}^{\infty} \exp(\pm i\alpha t^2) dt = \sqrt{\frac{\pi}{\mp i\alpha}} = \sqrt{\frac{\pm i\pi}{\alpha}} \quad (\text{Fresnel 積分})$$

$$\int_{-\infty}^{\infty} \exp(-\frac{a}{2}x^2) dx = \sqrt{\frac{2\pi}{a}}$$

ガウス積分

$$I_2 = \sqrt{\frac{i\pi}{\alpha}} \frac{1}{2} \sqrt{\frac{-i\pi}{\alpha}} = \frac{\pi}{2\alpha}$$

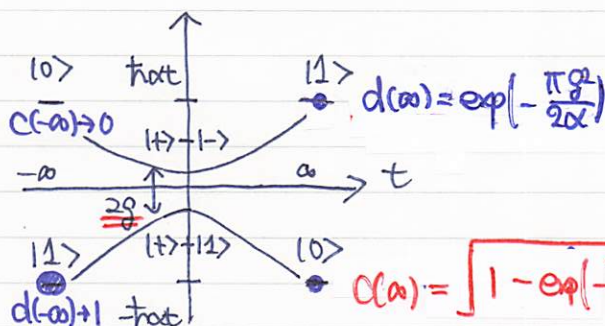
$$I_1 = \int_{-\infty}^{\infty} \exp(i\alpha t_1^2) \int_0^{t_1} \exp(-i\alpha t_2^2) dt_2 dt_1$$

$$\int_{-\infty}^{\infty} \exp(i\alpha t_1^2) \int_0^{t_1} \exp(-i\alpha t_2^2) dt_2 dt_1 = -I_2 = 0$$

$$\text{よ} \quad d(t \rightarrow \infty) = \exp(-\frac{\pi g^2}{2\alpha})$$

$$\textcircled{11} \quad d(-\infty) \rightarrow 1 \text{ なる } d(\infty) \text{ へ}$$

$C(\infty) \rightarrow 1$



$$\textcircled{11} \quad g^2 \text{ により } I \text{ が増える方が大なり小なり遷移!}$$

$$\textcircled{11} \quad g \text{ (大) なる方が } I \text{ が増える!}$$

$$\textcircled{11} \quad g=0 \text{ なる遷移なし!}$$

Stepup Landau-Zener 遷移

$$(1) \quad |1\rangle \sim \exp(-\frac{\pi g^2}{\alpha}) \text{ 遷移}$$

遷移

$$(1) \sim 1 - \exp(-\frac{\pi g^2}{\alpha}) \text{ 遷移}$$

$$\textcircled{11} \quad g: \text{ 相互作用で } I \text{ が増える!}$$

$$\textcircled{11} \quad \alpha: \text{ スピードが } I \text{ が増える!}$$

$$\textcircled{11} \quad \alpha < 1 \text{ なる方が } I \text{ が増える!}$$

断続的発展!

断続的時間発展 [$\hat{H}(t)$ がゆっくり変化] $\frac{d\hat{H}}{dt} \ll 1$

★準備

$$\hat{H}|u_n\rangle = E_n|u_n\rangle \quad \text{① 固有状態の正交展開}$$

Step up 固有状態展開

$$\begin{aligned} \hat{H}|u_n\rangle &= E_n|u_n\rangle && \text{直交} \\ \Rightarrow |\psi\rangle &= \sum_n C_n|u_n\rangle && \langle u_m|u_n\rangle = \delta_{m,n} \\ &&& \text{完全性} \\ &&& \sum_n |u_n\rangle\langle u_n| = I \end{aligned}$$

例 $f(x) = \sum a_k e^{ikx}$ (連続)

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \hat{f}(k) e^{ikx} \quad (\text{自由})$$

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dp \, e^{ipx/\hbar} \hat{\psi}(p) \quad (\hbar k = p)$$

対偶 $\hat{\psi}(p) = \int_{-\infty}^{\infty} dx \, e^{-ipx/\hbar} \psi(x)$

$$|\psi\rangle = \int_{-\infty}^{\infty} dp \, |p\rangle \langle p|\psi\rangle$$

② a 意味 $|p\rangle$ の展開

$$= \sum_n |u_n\rangle \langle u_n|\psi\rangle$$

③ a 意味 $|u_n\rangle$ の展開

$$\Rightarrow |\psi\rangle = \sum_n C_n |u_n\rangle \quad \text{④ } i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H}|\psi\rangle \quad \text{Schrödinger equation}$$

$$|\psi(t)\rangle = \sum_n C_n(t) |u_n\rangle \quad \text{Schrödinger equation}$$

⑤ $i\hbar \sum_n \frac{\partial}{\partial t} C_n(t) |u_n\rangle$

$$C_n(t) = C_n(0) \exp(-i \frac{1}{\hbar} E_n t)$$

⑥ $\sum_n C_n(t) E_n(t) |u_n\rangle$

★ $\hat{H} + \hat{H}(t)$ はなぜ...?

$$\hat{H}(t) |u_n(t)\rangle = E_n(t) |u_n(t)\rangle \quad \text{① 固有状態の正交展開}$$

$$|\psi(t)\rangle = \sum_n C_n(t) \exp(-i \frac{1}{\hbar} \int_0^t E_n(t') dt') |u_n(t)\rangle \quad \text{② } i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle$$

左 $\langle u_m(t)|$

③ $i\hbar \sum_n \left[\frac{\partial C_n}{\partial t} \exp(-i \frac{1}{\hbar} \int_0^t E_n(t') dt') |u_n(t)\rangle + C_n(t) \exp(-i \frac{1}{\hbar} \int_0^t E_n(t') dt') \frac{\partial}{\partial t} |u_n(t)\rangle \right] + \sum_n C_n(t) E_n(t) \exp(-i \frac{1}{\hbar} \int_0^t E_n(t') dt') |u_n(t)\rangle$

④ $\sum_n C_n(t) E_n(t) \exp(-i \frac{1}{\hbar} \int_0^t E_n(t') dt') |u_n(t)\rangle$

$$\therefore \frac{\partial}{\partial t} C_m(t) = - \sum_n C_n(t) \exp\left(i \frac{1}{\hbar} \int_0^t dt' (E_m(t') - E_n(t'))\right) \langle U_m(t) | \frac{\partial}{\partial t} | U_n(t) \rangle$$

① $\langle U_m(t) | \frac{\partial}{\partial t} | U_n(t) \rangle$ ではない?

② $\hat{H}(t)$ がわかりません
 $\frac{d\hat{H}}{dt}$ を考えよ!

① $\hat{H}(t) | U_n(t) \rangle = E_n(t) | U_n(t) \rangle$

(*)

$$\frac{d\hat{H}}{dt} | U_m(t) \rangle + \hat{H}(t) \frac{d}{dt} | U_m(t) \rangle = \frac{dE_m}{dt} | U_m(t) \rangle + E_m(t) \frac{d}{dt} | U_m(t) \rangle$$

左の $\langle U_m(t) |$ ($m \neq n$)

$$\langle U_m(t) | \frac{d\hat{H}}{dt} | U_m(t) \rangle + \langle U_m(t) | \hat{H}(t) \frac{d}{dt} | U_m(t) \rangle = E_m(t) \langle U_m(t) | \frac{d}{dt} | U_m(t) \rangle$$

$\langle U_m(t) | E_m(t) \rangle$
 $\frac{d\hat{H}}{dt}$ は $\frac{d}{dt}$ の $\frac{d}{dt}$ ではない

$$\therefore \langle U_m(t) | \frac{d\hat{H}}{dt} | U_m(t) \rangle = - (E_m(t) - E_n(t)) \langle U_m(t) | \frac{d}{dt} | U_m(t) \rangle$$

よ $\frac{d\hat{H}}{dt} \rightarrow 0$ なら $\langle U_m(t) | \frac{d}{dt} | U_m(t) \rangle \rightarrow 0$ ① $m \neq n$ は 0 になる

$$\frac{\partial}{\partial t} C_m(t) = - C_m(t) \langle U_m(t) | \frac{d}{dt} | U_m(t) \rangle$$

よ $C_m(t) = C_m(0) \exp\left(-i \int_0^t dt' \langle U_m(t') | \frac{d}{dt'} | U_m(t') \rangle\right)$

② $\langle U_n(t) | U_n(t) \rangle = 1$

$$\left(\frac{d}{dt} \langle U_n(t) | \right) | U_n(t) \rangle + \langle U_n(t) | \frac{d}{dt} | U_n(t) \rangle = 0$$

$\left(\langle U_n(t) | \frac{d}{dt} | U_n(t) \rangle \right)^*$

① $\frac{d}{dt} = 0$ なら $\frac{d}{dt}$ は $\frac{d}{dt}$ の $\frac{d}{dt}$ ではない

② $| C_m(t) | = | C_m(0) |$ の証明

Step up 断絶的発展

$\frac{d\hat{H}}{dt} \rightarrow 0$ なら $| C_m(t) | = | C_m(0) |$

① 基底状態が $\frac{d}{dt}$ に対して

⇒ Step up 量子 $P = -1/2$ $f(t) = 0 \Rightarrow 1$

$$\hat{H}(t) = f(t) \left(-\sum_j J_j \hat{\sigma}_j^z \hat{\sigma}_{j+1}^z - \sum_i h_i \hat{\sigma}_i^z \right) + (1-f(t)) \sum_i \hat{\sigma}_i^x$$

$\hat{H}_0$ $\hat{H}_1$

$\frac{d\hat{H}}{dt} \rightarrow 0$ なら

$\hat{H}_1$ の基底状態が

$\hat{H}_0$ の基底状態が

easy!

Hard

① $\frac{d}{dt} < 3$ の $\frac{d}{dt}$ ではない?

★ $C_{nm}(t) = |C_{nm}(t)|$ $C_{nm}(t) = 0$ となる条件は?

$$\frac{dC_{nm}}{dt} = -\sum_n C_{nm}(t) \exp\left(i \frac{1}{\hbar} \int_0^t \Delta_{nm}(t') dt'\right) \frac{\langle U_m(t) | \frac{dH}{dt} | U_m(t) \rangle}{\Delta_{nm}(t)} \quad (1) \quad C_{nm}(t) \neq 0$$

$$\Rightarrow \frac{d}{dt} C_{nm} = -\exp\left(i \frac{1}{\hbar} \int_0^t \Delta_{nm}(t') dt'\right) \frac{\langle U_m(t) | \frac{dH}{dt} | U_m(t) \rangle}{\Delta_{nm}(t)} \quad (1) \quad \int dt \text{ する} \\ \Delta_{nm}(t) \text{ は } \oplus \text{ かつ } \ominus \quad \uparrow \quad \text{でも面倒!}$$

(1) t に関する最大と最小を

$$C_{nm}(t) = \int_0^t dt_1 \exp\left(i \frac{1}{\hbar} \int_0^{t_1} \Delta_{nm}(t_2) dt_2\right) \frac{\langle U_m(t_1) | \frac{dH}{dt} | U_m(t_1) \rangle}{\Delta_{nm}(t_1)} \\ = -i \frac{1}{\hbar} \frac{\langle U_m | \frac{dH}{dt} | U_m \rangle}{\Delta_{nm}} \left[\exp\left(\frac{i}{\hbar} t \Delta_{nm}\right) - 1 \right] \quad (1) \quad \begin{matrix} \text{max} \\ \text{min} \end{matrix}$$

$$|C_{nm}(t)| = \hbar \frac{1}{|\Delta_{nm}|^2} \left| \langle U_m | \frac{dH}{dt} | U_m \rangle \right| \left| 2 \sin\left(\frac{\Delta_{nm} t}{2\hbar}\right) \right|$$

$$\begin{aligned} |e^{i0} - 1|^2 &= (e^{i0} - 1)(e^{-i0} - 1) \\ &= 2 - e^{i0} - e^{-i0} \\ &= 2(1 - \cos 0) \\ &= 4 \sin^2 \frac{0}{2} \end{aligned}$$

Step up Q.A. 成り立つ条件

$$\Delta_{nm}(t) = E_n(t) - E_m(t) \quad \text{が } \text{I 相対的} \text{ かつ } \text{II 相対的}$$

$$\tau \Delta_{nm}^2 \gg 1 \quad \text{かつ } \tau \gg \frac{1}{\Delta_{nm}^2} \quad \text{を満たす}$$

四 1次元-スピン

★ 2次元-スピン問題 [J. Roland and N. Carf: Phys. Rev. A 65 042308 (2002)]

N のうち 1 を指定した状態 $|n\rangle$ を選ぶ $\rightarrow O(N)$ 時間.

$$\hat{H}_0 = \hat{I} - |n\rangle\langle n| \quad \begin{cases} \hat{H}_0 |n\rangle = 0 \\ \hat{H}_0 |k\rangle = |k\rangle \quad (n \neq k) \end{cases} \quad \textcircled{11} \quad \boxed{\hat{H}_0 \text{ a G.S. is } |n\rangle}$$

$$\hat{H}_1 = \hat{I} - |\psi\rangle\langle\psi| \quad \begin{cases} \hat{H}_1 |\psi\rangle = 0 \\ \hat{H}_1 |\phi\rangle = |\phi\rangle \quad (\phi \neq 0) \end{cases} \quad \textcircled{11} \quad \boxed{\text{重畳した G.S.}}$$

$$\begin{cases} |\psi\rangle = \frac{1}{\sqrt{N}} \sum_k |k\rangle \quad (\phi=0) & \textcircled{11} \quad \langle\psi|\phi\rangle = \frac{1}{N} \sum_k \exp(i \frac{2\pi}{N} k \phi) \langle k|l\rangle = \delta_{kl} \\ |\phi\rangle = \frac{1}{\sqrt{N}} \sum_k \exp(i \frac{2\pi}{N} k \phi) |k\rangle & = \frac{1}{N} \sum_k \exp(i \frac{2\pi}{N} k \phi) = 0. \\ & 1 + \omega^k + \dots + \omega^{(N-1)k} = \frac{1 - \omega^{Nk}}{1 - \omega} \quad (\phi \neq 0) \end{cases}$$

行列表示 $\langle k|\hat{H}_0|l\rangle \rightarrow A_{kl}$

$\langle k|\hat{H}_1|l\rangle \rightarrow B_{kl}$

$$\Rightarrow \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} \quad \begin{pmatrix} 1 - \frac{1}{N} & & -\frac{1}{N} \\ & 1 - \frac{1}{N} & \\ -\frac{1}{N} & & 1 \end{pmatrix}$$

★ $|n\rangle$ を除外して考える

$$|\bar{n}\rangle = \frac{1}{\sqrt{N-1}} \sum_{k \neq n} |k\rangle$$

行列表示 $\begin{pmatrix} \langle n|\hat{A}|n\rangle & \langle n|\hat{A}|\bar{n}\rangle \\ \langle \bar{n}|\hat{A}|n\rangle & \langle \bar{n}|\hat{A}|\bar{n}\rangle \end{pmatrix}$

$$\hat{I} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad |n\rangle\langle n| \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\langle n|\psi\rangle = \frac{1}{\sqrt{N}}$$

$$\langle \bar{n}|\psi\rangle = \frac{1}{\sqrt{N}\sqrt{N-1}} \sum_{k \neq n} \langle k|l\rangle$$

$$|\psi\rangle\langle\psi| \rightarrow \begin{pmatrix} \frac{1}{N} & \frac{\sqrt{N-1}}{N} \\ \frac{\sqrt{N-1}}{N} & 1 - \frac{1}{N} \end{pmatrix}$$

$$= \frac{1}{\sqrt{N}\sqrt{N-1}} \left\{ \sum_k \left(\sum_l \langle k|l\rangle \right) - \sum_l \langle n|l\rangle \right\}$$

$$= \frac{1}{\sqrt{N}\sqrt{N-1}} (N-1) = \frac{\sqrt{N-1}}{\sqrt{N}}$$

$$\hat{H}(t) = f(t) \hat{H}_0 + (1-f(t)) \hat{H}_1$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & f \end{pmatrix} + (1-f) \begin{pmatrix} 1 - \frac{1}{N} & -\frac{\sqrt{N-1}}{N} \\ -\frac{\sqrt{N-1}}{N} & \frac{1}{N} \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} (1-f)(1 - \frac{1}{N}) & -(1-f)\frac{\sqrt{N-1}}{N} \\ -(1-f)\frac{\sqrt{N-1}}{N} & f + (1-f)\frac{1}{N} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{\Delta}{2} \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

$$\frac{1}{N} + f(1 - \frac{1}{N}) = \frac{1}{2} \quad \boxed{(1-f)(1 - \frac{1}{N})}$$

$$\Delta = \sqrt{(1-2(1-f)(1-\frac{1}{N}))^2 + 4(1-f)^2 \frac{N-1}{N^2}} = \sqrt{1-4(1-f)f(1-\frac{1}{N})}$$

$$\begin{cases} \cos \theta = \frac{1}{\Delta} \left\{ 1-2(1-f)(1-\frac{1}{N}) \right\} \\ \sin \theta = \frac{1}{\Delta} 2(1-f) \frac{N-1}{N} \end{cases}$$

★ 行列対角化

$$|\cos \theta \hat{G}_z + \sin \theta \hat{G}_x - \lambda \hat{I}| = \begin{vmatrix} \cos \theta - \lambda & \sin \theta \\ \sin \theta & -\cos \theta - \lambda \end{vmatrix} = \lambda^2 - 1 = 0 \quad \therefore \lambda = \pm 1$$

$$\hat{H}(t) |u_0(t)\rangle = \left(\frac{1}{2} - \frac{\Delta}{2} \right) |u_0(t)\rangle$$

$$\therefore \Delta(t) = E_1(t) - E_0(t) = \Delta$$

$$\hat{H}(t) |u_1(t)\rangle = \left(\frac{1}{2} + \frac{\Delta}{2} \right) |u_1(t)\rangle$$

★ 断熱条件 $\frac{df}{dt}(\hat{H}_0 - \hat{H}_1)$

$$\frac{1}{|\Delta|^2} |\langle u_1 | \frac{d\hat{H}}{dt} | u_0 \rangle| \ll 1$$

$$\text{よ} \left| \frac{df}{dt} \right| = \varepsilon \frac{\Delta^2}{|\langle u_1 | \hat{H}_0 - \hat{H}_1 | u_1 \rangle|} \equiv \alpha \Delta^2 \quad \text{①} \quad \left[\frac{df}{dt} \text{ は 時間増加率} \right]$$

$$\frac{df}{dt} = \alpha \left(\frac{1}{1-\frac{1}{N}+\frac{1}{N}} - 4f(1-f)(1-\frac{1}{N}) \right) = \frac{\varepsilon}{N} \left(1 + \frac{1-2f}{1-\frac{1}{N}+\frac{1}{N}} \right)$$

$$\int_0^1 \frac{df}{1 + \frac{(1-2f)^2}{(N-1)(2f-1)^2}} = \int_0^T \frac{\varepsilon}{N} dt \quad 2\sqrt{N-1} df = \frac{d\phi}{\cos^2 \phi}$$

$$\begin{cases} \tan \phi_0 = \sqrt{N-1} (-1) \\ \tan \phi_1 = \sqrt{N-1} (+1) \end{cases} \quad \therefore \phi_0 = -\phi_1$$

$$\frac{1}{2\sqrt{N-1}} \int_{-\phi_1}^{\phi_1} \frac{\cos^2 \phi}{\cos^2 \phi} d\phi = \frac{\varepsilon}{N} T \quad \text{よ} \quad T = \frac{N}{\varepsilon \sqrt{N-1}} \phi_1 \rightarrow \sqrt{N} \quad (N \rightarrow \infty)$$

$$\text{②} \quad [O(N) \text{ ではなく } O(\sqrt{N}) \text{ だ!}]$$

QA4 収束定理 (Morita and Nishimori (2006))

$$|C_{nm}(t)| = \frac{t}{|\Delta_{nm}|^2} \left| \langle u_m | \frac{d\hat{H}}{dt} | u_n \rangle \right| \left| 2 \sin\left(\frac{\Delta_{nm}}{2} t\right) \right| \ll 1$$

★ QA4-系論

$$\Delta_{mn}(t) = E_m(t) - E_n(t) \quad \textcircled{11} \quad \text{各時刻の固有値を考慮?}$$

難... 定理の前提!

$$\hat{H}(t) = \hat{H}_0 + \Gamma(t) \hat{H}_1 \quad \Gamma(t) = \Gamma_0 \rightarrow 0 \quad \frac{d\Gamma}{dt} < 0 \quad (\text{単調減少})$$

$$\begin{cases} \hat{H}_0 = -\sum_j J_j \hat{\sigma}_j^z \hat{\sigma}_j^z \text{ etc} \\ \hat{H}_1 = -\sum_i \hat{\sigma}_i^x \quad (\text{横磁場}) \end{cases}$$

$$\textcircled{\star} \left| \langle u_m | \frac{d\hat{H}}{dt} | u_0 \rangle \right| = -\frac{d\Gamma}{dt} \left| \langle u_m | \sum_{i=1}^N \hat{\sigma}_i^x | u_0 \rangle \right|$$

$$\begin{cases} |u_m(t)\rangle = \sum_k C_{mk}(t) |k\rangle \rightarrow \hat{\sigma}_i^x |u_m(t)\rangle = \sum_k C_{mk}(t) |k\rangle \\ \langle u_m(t) | \hat{\sigma}_i^x | u_0(t) \rangle = \cos\theta \end{cases}$$

$$\leq -N \frac{d\Gamma}{dt} \quad (\text{交換4枚})$$

★ Hopf の定理

$$\hat{M} = \text{非負値行列} \Rightarrow |\varepsilon_i| \leq \frac{k-1}{k+1} \varepsilon_0 \quad (\varepsilon_i = \text{固有値}) \quad \textcircled{11} \quad \hat{M} \text{ 作用!}$$

$$k = \max_{i,j,k} \left\{ \frac{M_{jk}}{M_{ik}} \right\} \leq \frac{\max_{ij} \{M_{ij}\}}{\min_{ij} \{M_{ij}\}} \quad \textcircled{11} \quad M_{ij} \text{ a min} \geq \max \varepsilon \text{ 考慮.}$$

$$\Rightarrow \hat{M} = (E_+ \hat{I} - \hat{H}(t))^N \Rightarrow E_+ \geq E_0^{\max} + \Gamma_0 \quad \textcircled{11} \quad \hat{H}(t) \text{ a max} \varepsilon \text{ 考慮}$$

$$E_+ \hat{I} - \hat{H}(t) = E_+ \hat{I} - \hat{H}_0 + \Gamma(t) \sum_i \hat{\sigma}_i^x$$

$$\textcircled{11} \quad M_{ij} \text{ a } \varepsilon \in \mathbb{R} \Rightarrow \text{非対角のみ}, M_{ij} \text{ a } \varepsilon \in \mathbb{C} \Rightarrow \text{対角} \varepsilon.$$

$$\max_{ij} \{M_{ij}\} \leq \frac{(E_+ - E_0^{\max} + N\Gamma_0)^N}{\text{差が大き!} \quad \text{交換4枚}}$$

$$\begin{aligned}
 \min_{i,j} [M_{ij}] &= \langle \phi | (P(t) \sum_{i=1}^N \hat{\sigma}_i^x)^N | \phi \rangle \quad \text{① } k=0 \text{ が最小値!} \\
 &= (P(t))^N \langle \phi | (\sum \hat{\sigma}_i^x) (\sum \hat{\sigma}_i^x) \dots (\sum \hat{\sigma}_i^x) | \phi \rangle \quad \text{② } N \text{ 回反転で得られ、場合分け} \\
 &= (P(t))^N \underbrace{N!}_{1 \sim N \text{ まで並べ}} \quad \text{③ } N \text{ 回反転で得られ、場合分け}
 \end{aligned}$$

$$\therefore k \leq \frac{(E_+ - E_0^{\min} + P_0 N)^N}{N! (P(t))^N} \sim \frac{1}{\sqrt{2\pi N}} \left(\frac{E_+ - E_0^{\min} + P_0 N}{N P(t)} \right)^N e^N$$

Step up Stirling's formula

$$N! \sim \sqrt{2\pi N} N^N e^{-N}$$

$$\text{Hopf} \quad |\varepsilon_i| \leq \frac{k-1}{k+1} \varepsilon_0 \quad (\varepsilon_i \text{ は } \hat{M}_a \text{ の固有値})$$

$$(E_+ - \lambda_i)^N \leq \frac{k-1}{k+1} (E_+ - \lambda_0)^N \quad \text{① } E_+ - \lambda_i > 0?$$

$$E_+ - \lambda_i \leq \left(\frac{k-1}{k+1} \right)^{1/N} (E_+ - \lambda_0)$$

$$\left[\left(\frac{k-1}{k+1} \right)^{1/N} \right]^N = \left(1 - \frac{2}{k+1} \right)^N \stackrel{27D-112}{=} 1 - \frac{2}{N(k+1)} + \dots \leq 1 - \frac{2}{N(k+1)}$$

$$\leq (E_+ - \lambda_0) - \frac{2}{N(k+1)} (E_+ - \lambda_0)$$

$$\text{よ} \quad \lambda_i - \lambda_0 \geq \frac{2}{N(k+1)} (E_+ - \lambda_0) \sim \frac{2}{Nk} (E_+ - \lambda_0)$$

$$\geq \frac{2/\sqrt{2\pi N}}{Nek} (E_+ - \lambda_0) \left(\frac{N}{E_+ - E_0^{\min} + P_0 N} \right)^N P(t)^N$$

A

よ $|C_{\text{atom}}(t)| \ll 1$ と仮定

$$-N \frac{dP}{dt} \frac{1}{A^2 P(t)^{2N}} = \delta \ll 1$$

$$\text{② } -\frac{N}{A^2} \left[P(t)^{-2N+1} \right] \frac{1}{-2N+1} = \delta t + c$$

$$P(t) = \left(\frac{2N-1}{N} A^2 \right)^{\frac{1}{1-2N}} (\delta t + c)^{\frac{1}{1-2N}}$$

Step up QAA 最終評価

$$P(t) \sim t^{-\frac{1}{2N}} \sim \text{破綻は G.S. とおける!}$$

$$\text{③ } P(t) \sim (\delta t + c)^{\frac{1}{1-2N}} \sim \text{QAA!}$$