



量子 $T = -1/2\gamma$ では.

$$\begin{cases} \hat{H}_0 = - \sum_{ij} J_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z - \sum_{i=1}^N h_i \hat{\sigma}_i^z & (\text{target}) \\ \hat{H}_1 = - \Gamma \sum_{i=1}^N \hat{\sigma}_i^x \rightarrow \text{重畳含み世の状態} & (\text{driver}) \end{cases}$$

$$\hat{H}(t) = f(t) \hat{H}_0 + (1 - f(t)) \hat{H}_1$$

$$f(t) : 0 \rightarrow 1 \quad (0 \leq t \leq T)$$

$$t : 0 \rightarrow T$$

original

$$\begin{cases} \hat{H}(t) = \hat{H}_0 + f(t) \hat{H}_1 \\ f(t) : \infty \rightarrow 0 \end{cases}$$

$$\underline{\hat{H}_1 \text{ の g.s.}} \rightarrow \underline{\hat{H}_0 \text{ の g.s. へ}}$$

$$\bigotimes_{i=1}^N \frac{1}{\sqrt{2}} (| \uparrow \rangle + | \downarrow \rangle)$$

"難問"

★ Ising model

元は Heisenberg model の単純化

$$\hat{H} = -J \sum_{\langle ij \rangle} (\cancel{\hat{\sigma}_i^x \hat{\sigma}_j^x} + \cancel{\hat{\sigma}_i^y \hat{\sigma}_j^y} + \hat{\sigma}_i^z \hat{\sigma}_j^z)$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\hat{\sigma}_1, \hat{\sigma}_2] = \hat{\sigma}_1 \hat{\sigma}_2 - \hat{\sigma}_2 \hat{\sigma}_1 = 2i\hat{\sigma}_3 \quad \begin{aligned} |\uparrow\rangle &= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned}$$

$$\hat{\sigma}_2 |\uparrow\rangle = 1 |\uparrow\rangle \quad \hat{\sigma}_2 |\downarrow\rangle = -1 |\downarrow\rangle$$

$$\Rightarrow E(\vec{\sigma}) = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \quad (\sigma_i = \pm 1)$$

↑
 $\hat{\sigma}_2 \rightarrow \sigma = \pm 1$ に
 “古典スピン”系になる

☆ 中置法 $G \rightarrow P(G)$

① 確率分布を扱う

{ 熱中置法 $G \rightarrow G^A$ (対角項)

{ 量中置法 $G \rightarrow \hat{G}$ (非対角項)

非PIL中置法 $J \rightarrow J_{ij}$ (スピンガラス)

☆ 断熱定理

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle \quad (\text{Schrödinger 方程式})$$

$$\hat{H}(t) = f(t) \hat{H}_0 + (1-f(t)) \hat{H}_1$$

$$f(t) = \frac{t}{\tau} \equiv s \quad (\tau = \tau = 1/2 \text{ 時間})$$

$$i\hbar \frac{d}{ds} |\psi(s)\rangle = \tau \hat{H}(s) |\psi(s)\rangle \quad \textcircled{1} \quad \boxed{\tau \otimes \hat{H}(s) \otimes 1}$$

$$\boxed{\text{各時刻}} \quad \hat{H}(s) |j(s)\rangle = E_j(s) |j(s)\rangle \quad \textcircled{2}$$

$$|\psi(s)\rangle = \sum_j C_j(s) |j(s)\rangle \quad \text{展開}$$

$$[\text{もし } |j(s)\rangle \rightarrow |j\rangle \text{ なら } C_j(s) = C_j(0) \exp(-i \frac{1}{\hbar} \tau E_j)]$$

$$\textcircled{3} \quad C_j(s) \equiv \tilde{C}_j(s) \exp(-i \frac{1}{\hbar} \tau \underbrace{\int_0^s du E_j(u)}_{\phi_j(s)}) \quad \text{と置く}$$

$$\textcircled{4} : i\hbar \frac{d}{ds} |\psi(s)\rangle = i\hbar \sum_j \frac{d\tilde{C}_j}{ds} \exp(\dots) |j(s)\rangle$$

$$+ \sum_j \tilde{C}_j(s) \tau E_j(s) \exp(\dots) |j(s)\rangle$$

$$+ i\hbar \sum_j \tilde{C}_j(s) \exp(\dots) \frac{d}{ds} |j(s)\rangle$$

$$\textcircled{10} \quad \hat{H}_j(s) |k(s)\rangle = \sum_j \tilde{C}_j(s) \exp(-i \frac{\tau}{\hbar} E_j(s)) |j(s)\rangle$$

$$\langle k(s) | \text{在左} \rangle \quad (\langle k(s) | j(s) \rangle = \delta_{kj} \text{ 对})$$

$$\therefore \frac{d\tilde{C}_k}{ds} = - \sum_j \tilde{C}_j(s) \exp(i \frac{\tau}{\hbar} (\phi_k - \phi_j)) \langle k(s) | \frac{d}{ds} | j(s) \rangle$$

$$[\textcircled{11} \text{ 对 } \frac{d\hat{H}}{ds} |j(s)\rangle + \hat{H}(s) \frac{d}{ds} |j(s)\rangle = \frac{dE_j}{ds} |j(s)\rangle + E_j(s) \frac{d}{ds} |j(s)\rangle]$$

$$\begin{cases} \langle k(s) | \frac{d}{ds} | j(s) \rangle = \frac{1}{E_j(s) - E_k(s)} \langle k(s) | \frac{d\hat{H}}{ds} | j(s) \rangle & (j \neq k) \\ \langle k(s) | \frac{d}{ds} | k(s) \rangle = 0 & (\text{by } \langle k(s) | k(s) \rangle = 1) \end{cases}$$

$$\Rightarrow \frac{d\tilde{C}_k}{ds} = \sum_{j \neq k} \frac{e^{i \frac{\tau}{\hbar} (\phi_k - \phi_j)} \tilde{C}_j(s) \langle k(s) | \frac{d\hat{H}}{ds} | j(s) \rangle}{E_k(s) - E_j(s)}$$

$\Delta E_{kj}(s) = E_k(s) - E_j(s)$

$$\textcircled{12} \quad \text{QA} \quad \begin{cases} \tilde{C}_0(0) = 1 \\ \tilde{C}_k(0) = 0 \end{cases} \xrightarrow{\text{g.s. 附近}} \begin{cases} \tilde{C}_0(s) = 1 - o(1/\tau) \\ \tilde{C}_k(s) = o(1/\tau) \end{cases}$$

$$\tilde{C}_k(s) = \int_0^s du \frac{e^{i \frac{\tau}{\hbar} \Delta \phi_{k0}}}{\Delta E_{k0}(u)} \tilde{C}_0(u) \langle k(s) | \frac{d\hat{H}}{ds} | 0(u) \rangle$$

$1 - o(1/\tau)$

$$= \int_0^s du \boxed{e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}} \frac{1}{\Delta_{k_0}(u)} \langle k(u) | \frac{dH}{ds} | 0(u) \rangle + O(\frac{1}{\tau^2})$$

$$\uparrow$$

$$\boxed{\frac{\hbar}{i\tau\Delta_{k_0}} \frac{d}{du} e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}}$$

$$= \int_0^s du \frac{\hbar}{i\tau} \boxed{\frac{\langle k(u) | \frac{dH}{ds} | 0(u) \rangle}{\Delta_{k_0}(u)^2}} \frac{d}{du} (e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}) \equiv A(u)$$

$$= \frac{i\hbar}{\tau} [A(0) - A(s) e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}] + \frac{i\hbar}{\tau} \int_0^s du e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}} \left(\frac{d}{du} A(u) \right)$$

$$|\tilde{C}_k(s)| < \varepsilon \text{ なる } s.$$

$$\varepsilon > \frac{\hbar}{\tau} \max_s \left(\frac{1}{\Delta_{k_0}(s)^2} \left| \langle k(s) | \left(\frac{dH}{ds} \right) | 0(s) \rangle \right| \right) \quad \text{2732-146 f2}$$

$\Rightarrow 0 \text{ 付近は危ない}$

Step up 断熱条件

$$\tau \geq \frac{1}{\varepsilon \Delta_{k_0}(s^*)^2} \quad \langle \psi_0 | \psi_0 \rangle = 1$$

$$\textcircled{11} \rightarrow \frac{dH}{ds} = 0 \text{ 付近?}$$

$$\textcircled{12} \quad s^2 \hat{H}_0 + (1-s^2) \hat{H}_1$$

$$\Rightarrow 2s(\hat{H}_0 - \hat{H}_1)$$

鈴木 国田 (2005) $s=1$ 付近22

$$P_{ex} \sim \frac{1}{\tau^2} \Rightarrow \text{QAの残留エネルギーの予測 (断熱定理)}$$

☆ 藤田 西森の条件 $\frac{\hbar}{i\tau\Delta\epsilon_0} \frac{d}{du} (e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}})$ $-B(u)$ とおく

$$\tilde{C}_k(s) = \int_0^s du \underline{e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}} \boxed{\frac{1}{\Delta\epsilon_0(u)} \langle k(u) | \frac{dH}{ds} | 0(u) \rangle}$$

$$\left[B_{k+1}(u) = \frac{1}{i\Delta\epsilon_0(u)} \frac{d}{du} B_k(u) \text{ とおく } \left(B_1(u) = \frac{B(u)}{i\Delta\epsilon_0(u)} \right) \right]$$

$$\begin{aligned} &= -\frac{\hbar}{\tau} \int_0^s du B_1(u) \frac{d}{du} e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}} \underline{\frac{\hbar}{i\tau\Delta\epsilon_0} \frac{d}{du} e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}} \\ &= \frac{\hbar}{\tau} \left[\underbrace{B_1(0)}_{\leftarrow 0} - \underbrace{B_1(s)}_{\leftarrow 0} e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}} \right] + \frac{\hbar}{\tau} \int_0^s du \underline{e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}}} \underline{\frac{d}{du} B_1(u)}^{B_2(u)} \\ &= \left(\frac{\hbar}{\tau} \right)^2 \int_0^1 du B_2(u) \frac{d}{du} e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}} \\ &= \dots = \left(\frac{\hbar}{\tau} \right)^m \int_0^1 du \underline{B_m(u)} \frac{d}{du} e^{i\frac{\tau}{\hbar}\Delta\phi_{k_0}} \end{aligned}$$

Step up 藤田 西森の主張 2-16

$$s = \frac{t}{\tau} \text{ と } \frac{d^m H}{ds^m} \Big|_{s=0,1} = 0 \text{ とおく}$$

$$|\tilde{C}_k(1)|^2 \sim \frac{1}{\tau^{2(m+1)}}$$

★ counter-diabatic 項

理想 $|\psi(t)\rangle = e^{i\phi_n(t)} |n(t)\rangle$

① $|n(t)\rangle$ a 基

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}^P(t) |\psi(t)\rangle$$

② $\hat{H}^P(t) = \hat{H}(t) + \hat{H}_{CD}(t)$

$$\downarrow$$

$$- \hbar \frac{d\phi_n}{dt} e^{i\phi_n} |n(t)\rangle + i\hbar e^{i\phi_n} \frac{d}{dt} |n(t)\rangle$$

$$\therefore \hat{H}_{CD}(t) |n(t)\rangle = (-\hbar \frac{d\phi_n}{dt} - E_n(t)) |n(t)\rangle + i\hbar \frac{d}{dt} |n(t)\rangle$$

↑ 補正項!

$$\begin{cases} \langle n(t) | \hat{H}_{CD} | n(t) \rangle = -\hbar \frac{d\phi_n}{dt} - E_n(t) + i\hbar \langle n(t) | \frac{d}{dt} | n(t) \rangle \\ \langle m(t) | \hat{H}_{CD} | n(t) \rangle = i\hbar \langle m(t) | \frac{d}{dt} | n(t) \rangle \end{cases}$$

↑ $\delta \langle n | \hat{H}_{CD} | n \rangle = 0$!

↑ ϕ_n は 時間

$$\frac{d\phi_n}{dt} = -\frac{1}{\hbar} E_n(t) + i \langle n(t) | \frac{d}{dt} | n(t) \rangle \quad \delta \phi_n \text{ は 実数}$$

$$\begin{aligned} \text{52 } \hat{H}_{CD}(t) &= \sum_{m \neq n} |m(t)\rangle \langle m(t) | \hat{H}_{CD} | n(t) \rangle \langle n(t)| \\ &= i\hbar \sum_{m \neq n} |m(t)\rangle \langle m(t) | \frac{d}{dt} | n(t) \rangle \langle n(t)| \end{aligned}$$

⑤ $\langle m(t) | \frac{d}{dt} | n(t) \rangle = \frac{1}{E_n(t) - E_m(t)} \langle m(t) | \frac{d\hat{H}}{dt} | n(t) \rangle$

step up $\hat{H}_{CD}$ 再.

$$\hat{H}_{CD}(t) = i\hbar \sum_{m \neq n} \frac{\langle m(t) | \frac{d\hat{H}}{dt} | n(t) \rangle}{E_n(t) - E_m(t)} |m(t)\rangle \langle n(t)| \delta \phi_n \text{ 項}$$

★ 现实的右CD项

$$\langle m | \hat{H}_{\text{cd}} | n \rangle = i \frac{\langle m | \frac{d\hat{H}}{dt} | n \rangle}{E_n - E_m} \quad \begin{array}{l} \textcircled{11} \\ \text{↑} \end{array} \begin{array}{l} E_n - E_m \text{ in?} \\ \hat{H} \text{ ca 交换顺序} \end{array}$$

$$\langle m | [\hat{H}_{\text{cd}}, \hat{H}] | n \rangle = \langle m | \hat{H}_{\text{cd}} E_n - E_m \hat{H}_{\text{cd}} | n \rangle$$

$$\text{所以 } i \langle m | \frac{d\hat{H}}{dt} | n \rangle = \langle m | [\hat{H}_{\text{cd}}, \hat{H}] | n \rangle$$

$$G(\hat{H}_{\text{cd}}) \equiv \frac{d}{dt} \hat{H}(t) + i [\hat{H}_{\text{cd}}(t), \hat{H}(t)] \quad \begin{array}{l} \textcircled{11} \\ \text{↑} \end{array} \begin{array}{l} \text{可证 } \hat{H}_{\text{cd}} \text{ 厄米性} \\ \text{厄米性/选择也!} \end{array}$$

step up! Sel's-Polkovnikova 变分原理 (2017)

$$\min_A \{ \text{Tr} [G(A)^2] \} \quad G(A) = \frac{d}{dt} \hat{H}(t) + i [\hat{A}, \hat{H}]$$

$$\delta \hat{G} = i [\delta \hat{A}, \hat{H}]$$

$$\text{所以 } 2 \text{Tr} [\hat{G}(A) \delta \hat{G}] = 2i \text{Tr} [\hat{G}(A) [\delta \hat{A}, \hat{H}]] = 0 \quad \begin{array}{l} \text{理论} \\ \text{↑} \end{array}$$

step up! $\text{Tr} \in [n]$

$$\begin{aligned} \text{Tr} [\hat{A} \hat{B}] &= \text{Tr} [\hat{B} \hat{A}] \quad (\text{循环性}) \\ \text{Tr} [\hat{A} [\hat{B}, \hat{C}]] &= \text{Tr} [[\hat{C}, \hat{A}] \hat{B}] \\ \{ \text{Tr} [\text{ABC}] - \text{Tr} [\text{ACB}] \} \\ &\quad \text{Tr} [\text{CAB}] \end{aligned}$$

$$\therefore 2i \operatorname{Tr}[G[\delta A, \hat{H}]] = \underline{2i} \operatorname{Tr}[\underline{[\hat{H}, \hat{G}]}, \delta \hat{A}]$$

勾配

$$G(0) = \frac{d\hat{H}}{d\lambda} \text{ から変化する}$$

$$A = i[\hat{H}, \frac{d\hat{H}}{d\lambda}] \propto$$

但し $\nabla \times \nabla u = 0$ (誤)

$$\left[\frac{d\hat{H}}{d\lambda} = \frac{d}{d\lambda} (\lambda \hat{H}_0 + (1-\lambda) \hat{H}_1) = \hat{H}_0 - \hat{H}_1 \right]$$

$$\propto i[\hat{H}_0, \hat{H}_1]$$

(1) この用意で!

$$[\hat{\sigma}^z, \hat{\sigma}^x] = 2i\hat{\sigma}^y$$

量子断熱近似 PULJISZ (QAOA) (2014) (重松氏)

☆ 試行状態 (ansatz) $|s\rangle = \bigotimes_{i=1}^N \left(\frac{1}{\sqrt{2}} (|1\rangle + |0\rangle) \right)$

$|\vec{r}, \vec{\alpha}\rangle = \prod_{k=1}^P \left(e^{-i \underline{\alpha_k} \hat{H}_1} e^{-i \underline{r_k} \hat{H}_0} \right) |s\rangle$

$1^{\circ} 3 \times 2$

1/2
 $\hat{H}_0, \hat{H}_1$
 行列

$F_P(\vec{r}, \vec{\alpha}) = \langle \vec{r}, \vec{\alpha} | \hat{H}_0 | \vec{r}, \vec{\alpha} \rangle$

$\vec{r}, \vec{\alpha}$ を変化する F_P を Find
 勾配法など

☆ QA の 問題

$\hat{U}(\tau) = \exp\left(-i \frac{1}{\hbar} \int_0^\tau dt \hat{H}(t)\right)$

$= \exp\left(-i \frac{\Delta t}{\hbar} \sum_{k=1}^n \hat{H}_k\right)$ (ただし $\hat{H}_k = \hat{H}(k\Delta t)$)

Step up 鈴木 No.2 分解

$\Delta t = \frac{\tau}{n}$

$e^{t(\hat{A}+\hat{B})} \stackrel{1^{\circ}}{\approx} \left(e^{\frac{t}{n}\hat{A}} e^{\frac{t}{n}\hat{B}} \right)^n$ (Lie-Trotter)
 $\stackrel{2^{\circ}}{\approx} \left(e^{\frac{t}{2n}\hat{A}} e^{\frac{t}{n}\hat{B}} e^{\frac{t}{2n}\hat{A}} \right)^n$ (Suzuki-Trotter)
 etc 中物理

$$\textcircled{2} \quad e^{\delta(\hat{A}+\hat{B})} = \hat{I} + \delta(\hat{A}+\hat{B}) + \frac{\delta^2}{2} (\hat{A}+\hat{B})^2 + \dots$$

$\hat{A}^2 + \hat{A}\hat{B} + \hat{B}\hat{A} + \hat{B}^2$

$$e^{\delta\hat{A}} e^{\delta\hat{B}} = (\hat{I} + \delta\hat{A} + \frac{\delta^2}{2}\hat{A}^2) (\hat{I} + \delta\hat{B} + \frac{\delta^2}{2}\hat{B}^2)$$

$$\rightarrow = \hat{I} + \delta(\hat{A}+\hat{B}) + \frac{\delta^2}{2}\hat{A}^2 + \frac{\delta^2}{2}\hat{B}^2 + \delta\hat{A}\hat{B} + \dots$$

$$(\text{誤差}) = -\frac{\delta^2}{2} [\hat{A}, \hat{B}]$$

$$\text{no} \Rightarrow -\frac{\tau}{2} \left(\frac{\tau}{n} \right) [\hat{A}, \hat{B}] \text{ 分々23}$$

$$U(\tau) \approx \prod_{k=1}^n \exp(-i \frac{\Delta t}{\hbar} f_k \hat{H}_0) \exp(-i \frac{\Delta t}{\hbar} (1-f_k) \hat{H}_1)$$

$$[\hat{H}_k = f_k \hat{H}_0 + (1-f_k) \hat{H}_1]$$

$\prod_{k=1}^n$
 212-115 誤差
 kth order
 QA $\rightarrow$ OADA

$n \rightarrow \infty$ 212-115
有限誤差!

(NISQ)

★ 最適化-原理 (1956)

step up $g(t) =$ 制御入力の最適化

$$\frac{dx}{dt} = f(x, g) \text{ に対して } \min_g \{ F(x(T)) \} \text{ を求め}$$

$$\Rightarrow J = F(x(T)) + \int_0^T dt \underbrace{\lambda(t)}_{\text{costate (77222 未定変数)}} \left(\underbrace{f(x, g)}_{\text{各時刻 } t \text{ の } P(x, \lambda, g) \text{ を } \min \text{ する}} - \frac{dx}{dt} \right) \text{ を求め}$$

$$\begin{cases} \delta \lambda = \frac{d\lambda}{dt} = \frac{\partial P}{\partial x} (= f(x, g)) & \text{①} \quad \text{1211-2 方程式} \\ \delta x = \frac{d\lambda}{dt} = -\frac{\partial P}{\partial \lambda} & \text{②} \quad \lambda(t) \text{ に関する} \end{cases}$$

$$\delta x(t) = \frac{\lambda(t)}{\text{②}} = -\frac{\partial F(x(t))}{\partial x} \quad \text{① による}$$

$$\left[\text{by } \frac{\partial F}{\partial x} \delta x(t) - \int_0^T dt \left[\lambda(t) \left(-\frac{d}{dt} \delta x \right) + \frac{\partial F}{\partial x} \delta x \right] \right]$$

$$\left[\lambda(t) \delta x(t) \right]_0^T - \int_0^T dt \frac{d\lambda}{dt} \delta x(t)$$

$$\lambda(t) \delta x(t) - \lambda(0) \delta x(0)$$

$$\Rightarrow g^*(t) = \arg \min_g \{ P(x^*, \lambda^*, g) \} \text{ を求め}$$

QAOA

$$F_p(T) = \langle \vec{r}, \vec{\alpha} | \hat{H}_0 | \vec{\alpha}, \vec{r} \rangle \stackrel{\text{Costate}}{=} \min_{\vec{r}, \vec{\alpha}} U_{\text{cost}}$$

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle$$

$$(-i\hbar \frac{d}{dt} \langle \psi(t)| = \langle \psi(t)| \hat{H}(t))$$

$$(\langle \psi(t)| \hat{H}(t) + i\hbar \frac{d}{dt} \langle \psi(t)|) |\chi'(t)\rangle$$

$$\Rightarrow J = F_p(T) + \int_0^T dt \left[\underbrace{\langle \chi(t)| \hat{H} - i\hbar \frac{d}{dt} |\psi(t)\rangle}_{\text{Costate}} + \underbrace{\langle \psi(t)| \hat{H}(t) + i\hbar \frac{d}{dt} |\chi'(t)\rangle}_{\text{Value-Function}} \right]$$

$$\delta \langle \chi | = \langle \chi(t)| \hat{H} \oplus i\hbar \frac{d}{dt} \langle \chi(t)| = 0 \quad \Rightarrow \quad i\hbar \frac{d}{dt} |\chi(t)\rangle = \hat{H}(t) |\chi(t)\rangle$$

$$\delta \langle \chi | = (\hat{H} - i\hbar \frac{d}{dt}) |\psi(t)\rangle = 0 \quad \Rightarrow \quad i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle$$

đối xứng

$$\delta \langle \psi | = (\hat{H} \ominus i\hbar \frac{d}{dt}) |\chi(t)\rangle = 0 \quad \Rightarrow \quad i\hbar \frac{d}{dt} |\chi'(t)\rangle = \hat{H}(t) |\chi'(t)\rangle$$

$$\delta \langle \psi(t) | \cdot \hat{H}_0 |\psi(t)\rangle + i\hbar |\chi'(t)\rangle = 0 \quad \Rightarrow \quad |\chi'(t)\rangle \propto \hat{H}_0 |\psi(t)\rangle$$

$$P(|\psi\rangle, |\chi\rangle, f) = \langle \chi(t) | \hat{H}(t) | \psi(t) \rangle + \langle \psi(t) | \hat{H}(t) | \chi(t) \rangle$$

$$= 2\text{Re} \langle \chi(t) | \hat{H}(t) | \psi(t) \rangle \propto \max_{\vec{r}, \vec{\alpha}} \mathbb{E}_{\text{cost}}$$

$$\frac{\partial P}{\partial f} = 2\text{Re} \langle \chi(t) | \hat{H}_0 - \hat{H}_1 | \psi(t) \rangle \equiv \phi(t)$$

$$\phi(t) > 0 \quad \underline{f(t) \rightarrow 0} \quad \hat{H}_1 \text{ on?} \quad \phi(t) < 0 \text{ thì } \underline{f(t) \rightarrow 1} \quad \hat{H}_0 \text{ on?} \quad \text{bang-bang control}$$

量子モリタル法

★ $\beta \rightarrow \infty$ を調べる

① $\boxed{\text{QAの基底 g.s.}}$ ($\beta = \frac{1}{T}$)

$$P(\vec{\sigma}) = \frac{1}{Z} \exp(-\beta E(\vec{\sigma})) \rightarrow \hat{P} = \frac{1}{Z} \exp(-\beta \hat{H})$$

$$Z = \sum_{\vec{\sigma}} \exp(-\beta E(\vec{\sigma})) \quad Z = \text{Tr}[\exp(-\beta \hat{H})]$$

★ 横磁場の別表現

$$\hat{H}_1 = -J \sum_{i=1}^N \hat{\sigma}_i^x \quad e^{\beta(\hat{A}+\hat{B})} \approx (e^{\frac{\beta}{M}\hat{A}} e^{\frac{\beta}{M}\hat{B}})^M$$

$$\begin{aligned} Z &= \text{Tr} \left\{ \exp(-\beta(\hat{H}_0(\hat{\sigma}^z) + \hat{H}_1(\hat{\sigma}^x)) \right\} \quad \text{① 対角化が2倍} \\ &= \text{Tr} \left\{ \prod_{k=1}^M \exp(-\frac{\beta}{M}\hat{H}_0) \exp(-\frac{\beta}{M}\hat{H}_1) \right\} \quad \text{② Suzuki-Trotter?} \\ &\quad \underbrace{I = \sum_{\vec{\sigma}_k} |\vec{\sigma}_k\rangle\langle\vec{\sigma}_k|}_{\text{③ 演算子の間}} \\ &= \text{Tr} \sum_{\vec{\sigma}} \left\{ \prod_{k=1}^M \exp(-\frac{\beta}{M}H_0(\vec{\sigma}_k)) |\vec{\sigma}_k\rangle\langle\vec{\sigma}_k| \exp(-\frac{\beta}{M}\hat{H}_1) \right\} \quad \text{④ } \sum_{\vec{\sigma}} |\vec{\sigma}_k\rangle\langle\vec{\sigma}_k| = I \end{aligned}$$

⑤ $\hat{H}_1$ は横磁場

$$\exp\left(\frac{\beta J}{M} \hat{\sigma}_i^x\right) = \hat{I} + \frac{\beta J}{M} \hat{\sigma}_i^x + \frac{1}{2} \left(\frac{\beta J}{M}\right)^2 \hat{\sigma}_i^{xx} + \dots$$

$$= \cosh\left(\frac{\beta J}{M}\right) \hat{I} + \sinh\left(\frac{\beta J}{M}\right) \hat{\sigma}_i^x$$

$I = \sum_{\vec{\sigma}_k} |\vec{\sigma}_k\rangle\langle\vec{\sigma}_k|$
 前 $k=1$ の成分
 $|\vec{\sigma}_k\rangle\langle\vec{\sigma}_k|$

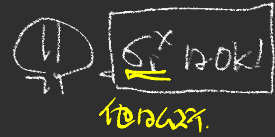
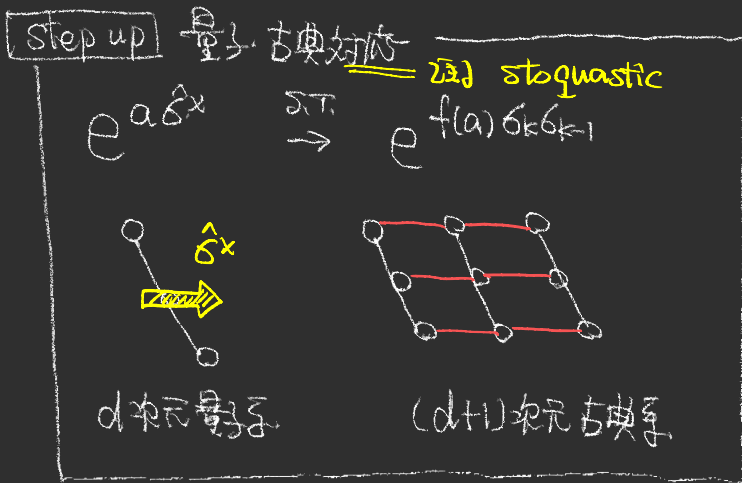
$$= \cosh\left(\frac{\beta P}{M}\right) \begin{pmatrix} 1 & \tanh\left(\frac{\beta P}{M}\right) \\ \tanh\left(\frac{\beta P}{M}\right) & 1 \end{pmatrix}$$

$$\left[e^{-2k} = \tanh\left(\frac{\beta P}{M}\right) \right] \begin{cases} P=0 \rightarrow \infty \\ k=\infty \leftarrow 0 \end{cases}$$

$$= \cosh\left(\frac{\beta P}{M}\right) e^{-k} \begin{pmatrix} e^{\overset{\text{red}}{\uparrow} k} & e^{\overset{\text{red}}{\downarrow} k} \\ \underbrace{e^{\overset{\text{red}}{\downarrow} k} & e^{\overset{\text{red}}{\uparrow} k}}_{\langle \sigma_k | e^{k \sigma_k \sigma_{k-1}} | \sigma_{k-1} \rangle} \end{pmatrix}$$

$$= \cosh\left(\frac{\beta P}{M}\right) e^{-k} \exp(k \sigma_k \sigma_{k-1})$$

$$\prod_{k=1}^M \exp\left(-\frac{\beta}{M} H_0(\vec{\sigma}_k) + k \sum_{j=1}^N \sigma_{jk} \sigma_{j,k-1}\right) = \text{古典2次元系?}$$



Non-stoguastic.

解析的 PPD-干

★ p-spin model

$$\hat{H}_0 = - \frac{J}{N^{p-1}} \sum_{i_1, i_2, \dots, i_p} \sigma_{i_1} \sigma_{i_2} \dots \sigma_{i_p} = -NJ \left(\frac{1}{N} \sum_i \sigma_i^2 \right)^p$$

$$Z = \text{Tr} [\exp(-\beta(\hat{H}_0 + \hat{H}_1))] \quad \left[K = -\frac{1}{2} \log \tanh\left(\frac{\beta J}{M}\right) \right]$$

$$\stackrel{\text{S.T.}}{=} \text{Tr} \sum_{\sigma} \prod_{k=1}^M \exp\left(-\frac{\beta}{M} H_0(\vec{\sigma}_k^2) + K \sum_{i=1}^N \sigma_{ik} \sigma_{i, k-1}\right)$$

同相约束

① 求迹计算!

$$= \sum_{\sigma} \prod_{k=1}^M \exp\left(\frac{NJ}{M} \left(\frac{1}{N} \sum_i \sigma_{ik} \right)^p + K \sum_{i=1}^N \sigma_{ik} \sigma_{i, k-1}\right)$$

→ m_k with $\int dm_k \delta(Nm_k - \sum_{i=1}^N \sigma_{ik})$

Step up 求和干①

$$1 = \int dm \delta\left(Nm - \sum_{i=1}^N \sigma_i\right)$$

$$\int d\tilde{m}_k \int dm_k \exp(-N\tilde{m}_k m_k + \tilde{m}_k \sum_i \sigma_{ik})$$

等式制约 $m = \frac{1}{N} \sum_{i=1}^N \sigma_i$

Step up 求和干②

$$\delta\left(Nm - \sum_{i=1}^N \sigma_i\right) = \int d\tilde{m} \exp(-\tilde{m}(Nm - \sum_i \sigma_i))$$

$$= \int \mathcal{D}m \int \mathcal{D}\tilde{m} \exp \left(\frac{N\beta J}{M} \sum_k m_k^p - N \sum_k m_k \tilde{m}_k \right)$$

$$\left(\sum_{\delta} \left(\prod_{k=1}^M \exp(\tilde{m}_k \delta_k + K \delta_k \delta_{k-1}) \right) \right)^N \quad \textcircled{11} \quad \begin{array}{l} \rightarrow \text{2D 磁場 } \tilde{m}_k \\ \text{1D Ising model} \end{array}$$

★ 鞍点近似 ($m_k \rightarrow m \quad \tilde{m}_k \rightarrow \frac{\beta}{M} \tilde{m}$)

$$\textcircled{\star} = \frac{M}{\pi} \exp \left(\frac{\beta \tilde{m}}{M} \delta_k + \boxed{K \delta_k \delta_{k-1}} \right) \quad \textcircled{11} \quad \begin{array}{l} \text{2D 磁場?} \end{array}$$

$$\rightarrow \text{Tr} \frac{M}{\pi} \exp \left(\boxed{\frac{\beta \tilde{m}}{M} \delta_k^2 + \frac{\beta P}{M} \delta_k^x} \right) = A$$

$$\hat{A} = \frac{\beta}{M} \begin{pmatrix} \hat{m} & P \\ P & -\hat{m} \end{pmatrix} \rightarrow \Lambda = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}$$

$$\det(\hat{A} - \lambda \hat{I}) = (\lambda - \tilde{m})(\lambda + \tilde{m}) - P^2 = 0$$

$$\therefore \lambda_{\pm} = \pm \beta \sqrt{\tilde{m}^2 + P^2} / M$$

$$= \text{Tr} (U \Lambda^M U^{-1}) = 2 \cosh \beta \sqrt{\tilde{m}^2 + P^2} \quad \square$$

$$Z = \int dm \int d\tilde{m} \exp(N(\beta J m^p - \beta m \tilde{m} + \ln 2 \cosh \beta \sqrt{\tilde{m}^2 + p^2}))$$

step up 鞍点法 ($N, \beta \rightarrow \infty$) $f(m, \tilde{m})$

$$I = \int dx \exp(+Nf(x)) \sim \exp(+Nf(x^*))$$

$$x^* = \max_x f(x) \quad \left(\frac{\partial f}{\partial x} = 0 \text{ 代入} \right)$$

★ 鞍点方程

$$\begin{cases} \partial m : \tilde{m} = p J m^{p-1} \\ \partial \tilde{m} : m = \frac{\tilde{m}}{\sqrt{\tilde{m}^2 + p^2}} \tanh \beta \sqrt{\tilde{m}^2 + p^2} \end{cases}$$

(11) $\beta \rightarrow \infty$ 则 $P \in \text{鞍点法 (QA)}$
 $\beta \rightarrow 0$ 则 $\beta \in \text{鞍点法 (SA)}$

★ QAOA

$$P_{\vec{\alpha}, \vec{r}}(\vec{\sigma}) = |\langle \vec{\sigma} | \vec{\alpha}, \vec{r} \rangle|^2$$

$$Z = \sum_{\vec{\sigma}} \int d\vec{\alpha} \int d\vec{r} \exp(-\beta E(\vec{\sigma})) P_{\vec{\alpha}, \vec{r}}(\vec{\sigma})$$

① $p=1$ 为例

$$\bigotimes_{i=1}^N \frac{1}{\sqrt{2}} (|1\rangle + |0\rangle)$$

$$|\alpha, r\rangle = \exp(-i\alpha \hat{H}_1) \exp(-ir \hat{H}_0) |S\rangle$$

$$= \left(\frac{1}{\sqrt{2}} \right)^N \sum_{\vec{\sigma}} \exp(-i\alpha \hat{H}_1) \exp(-ir H_0(\vec{\sigma}))$$

$$\underbrace{\left(\frac{1}{2\pi}\right)^N}_{p\text{-spm}} \int d\mathbf{m} \int d\tilde{\mathbf{m}} \sum_{\hat{\alpha}} \underbrace{\exp(-i\alpha \hat{H}_1)}_{\cos\alpha \frac{\hat{\sigma}^x}{2} + i\sin\alpha \frac{\hat{\sigma}^y}{2}} \exp(-iN\gamma m^p - N\mathbf{m}\tilde{\mathbf{m}} + \underbrace{\tilde{\mathbf{m}} \sum_{i=1}^N \hat{\sigma}_i}_{\cosh\tilde{\mathbf{m}} \frac{\hat{\sigma}^x}{2} + \sinh\tilde{\mathbf{m}} \frac{\hat{\sigma}^y}{2}})$$

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$$Z = \sum_{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2} \int d\mathbf{r} \int d\alpha \int d\mathbf{m}_0 \int d\mathbf{m}_1 \int d\mathbf{m}_2 \int d\tilde{\mathbf{m}}_0 \int d\tilde{\mathbf{m}}_1 \int d\tilde{\mathbf{m}}_2$$

$$\times \exp(N\beta J m_0^p - iN\gamma m_1^p + iN\gamma m_2^p \\ - N\mathbf{m}_0\tilde{\mathbf{m}}_0 - N\mathbf{m}_1\tilde{\mathbf{m}}_1 - N\mathbf{m}_2\tilde{\mathbf{m}}_2)$$

$$\left(\text{Tr} \left[B(\mathbf{r}, \mathbf{m}_2) A^\dagger(\alpha) e^{\tilde{\mathbf{m}}_0 \hat{\sigma}_2^x} \hat{A}(\alpha) \hat{B}(\mathbf{r}, \tilde{\mathbf{m}}_1) \right] \right)^N$$

虚時間量子 $\hbar=1$

$$\text{実時間} \quad i\hbar \frac{d}{dt} \psi(x,t) = \hat{H}(x,t) \psi(x,t) \quad \left[\hat{H}(x,t) = -\frac{\partial^2}{\partial x^2} + V(x) \right]$$

$$\boxed{t = -i\tau} \quad - \frac{d}{d\tau} \psi(x,\tau) = \hat{H}(x,\tau) \psi(x,\tau) \quad (\text{虚時間}) \quad \left[\begin{array}{l} \psi(x,\tau) \\ = \exp\left[-\int_0^\tau dt \hat{H}(x,t)\right] \psi(x,0) \end{array} \right]$$

forward

$$\left[- \frac{d}{d\tau} \tilde{\psi}(x,\tau) = -\hat{H}(x,\tau) \tilde{\psi}(x,\tau) \quad (\text{随伴}) \right]$$

backward/adjoint

$$\Rightarrow P(x,\tau) = \psi(x,\tau) \tilde{\psi}(x,\tau) \quad \textcircled{11} \quad \boxed{\text{2つの方程式に従う?}}$$

$$\frac{\partial P}{\partial \tau} = \underbrace{\frac{\partial \psi}{\partial \tau}}_{-H\psi} \tilde{\psi} + \psi \underbrace{\frac{\partial \tilde{\psi}}{\partial \tau}}_{H\tilde{\psi}} = -\left(\frac{\partial \psi}{\partial x^2} \tilde{\psi} - \psi \frac{\partial \tilde{\psi}}{\partial x^2} \right)$$

$J(x,\tau) = \psi \frac{\partial \tilde{\psi}}{\partial x} - \tilde{\psi} \frac{\partial \psi}{\partial x}$

$$= -\frac{\partial}{\partial x} \left(-\frac{\partial \psi}{\partial x} \tilde{\psi} + \psi \frac{\partial \tilde{\psi}}{\partial x} \right) \quad \textcircled{12} \quad \boxed{\text{Fokker-Planck方程式?}}$$

step up 7.4.1-7.4.2 方程式

$$\frac{\partial}{\partial t} P(x,t) = -\frac{\partial}{\partial x} J(x,t) \quad (\text{連続の方程式})$$

$$\left[J(x,t) = -\frac{\partial P}{\partial x} + f(x,t) P(x,t) \right]$$

⑬ $f(x,t) = -\frac{\partial V}{\partial x}$

$\frac{\partial P}{\partial t} = 0$ (定常)

$J(x,t) = 0$ (平衡)

$P(x) \propto \exp(-\frac{1}{\hbar} V(x))$

対応する Langevin 方程式

$$\left[dx = f(x,t) dt + \sqrt{2D} dW \right]$$

$$\star \psi = P/\tilde{\psi} \quad \left(\frac{\partial \psi}{\partial x} = \frac{1}{\tilde{\psi}} \frac{\partial P}{\partial x} - \frac{P}{\tilde{\psi}^2} \frac{\partial \tilde{\psi}}{\partial x} \right)$$

$$\frac{\partial}{\partial \tau} P(x, \tau) = \rightarrow \frac{\partial^2 P}{\partial x^2} - \underbrace{\left(\frac{\partial}{\partial x} \right)}_{\tilde{\psi}} P(x, \tau) \underbrace{\left(\frac{\partial}{\partial x} \right)}_{f_+(x, \tau)} \tilde{\psi} \quad (\text{forward FP})$$

$$\star \tilde{\psi} = P/\psi \quad \left(\frac{\partial \tilde{\psi}}{\partial x} = \frac{1}{\psi} \frac{\partial P}{\partial x} - \frac{P}{\psi^2} \frac{\partial \psi}{\partial x} \right)$$

$$\frac{\partial}{\partial \tau} P(x, \tau) = -\rightarrow \frac{\partial^2 P}{\partial x^2} - \underbrace{\left(\frac{\partial}{\partial x} \right)}_{\tilde{\psi}} P(x, \tau) \underbrace{\left(\frac{\partial}{\partial x} \right)}_{f_-(x, \tau)} \tilde{\psi} \quad (\text{backward FP})$$

$$\left[\begin{array}{l} \text{好底相33312} \\ \text{or} \end{array} \quad \begin{array}{l} dx = f_-(x, \tau) dt + \sqrt{2D} dW \quad (dt < 0) \\ dy = -f_-(y, T-s) ds + \sqrt{2D} dW \end{array} \right]$$

★ $T=72114$ 11111分解

$$\left(-\frac{\partial^2}{\partial x^2} + V(x) \right) \psi(x) = E \psi(x)$$

① 1階 ⊕ 容易

Step up $T=72114$ 11111分解

$$\hat{A} = \frac{\partial}{\partial x} + \frac{1}{2} V'(x)$$

$$\hat{A}^+ = -\frac{\partial}{\partial x} + \frac{1}{2} V'(x)$$

$$\hat{A} \psi(x) = 0 \text{ 有, } \psi(x) \propto \exp\left(-\frac{1}{2} V(x)\right)$$

$$\begin{aligned} \hat{A}^+ \hat{A} &= \left(-\frac{\partial}{\partial x} + \frac{1}{2} V'(x) \right) \left(\frac{\partial}{\partial x} + \frac{1}{2} V'(x) \right) \\ &= -\frac{\partial^2}{\partial x^2} + \underbrace{\left(\frac{1}{4} (V'(x))^2 - \frac{1}{2} V''(x) \right)}_{\text{位置I和II-11合}} \equiv \hat{H}_{\text{susy}} \end{aligned}$$

Step up SUSY 13111122

$$\hat{H}_{\text{SUSY}} = \hat{A}^\dagger \hat{A} \quad \left(\hat{A} = \frac{\partial}{\partial x} + \frac{1}{2} V(x) \right)$$

$$\hat{H}_{\text{SUSY}} \psi(x) = 0 \quad \psi(x) \propto \exp\left(-\frac{1}{2} V(x)\right)$$

① $2\alpha V(x)$ は
Feynman 2重積分

② L が
g.s.f

③ $V(x) = \frac{1}{2} x^2$ ($V'(x) = x$) 調和振動子

☆ 拡散モデル $P_0(x)$: 一々分布 $\xrightarrow[\text{生成}]{\text{拡散}}$ $P_T(x)$: 作りかた分布
(状態) $V(x)$ を与える? (例) Γ の分布 (基底状態)

④ $P(x, t) = \psi_t(x) \hat{\psi}_t(x)$

① $\hat{A}^\dagger \hat{A}$ は $\frac{\partial}{\partial x} p(x, t)$
一々分布を再現

$$\psi_t(x) = \sqrt{P(x, t)} \quad \text{を g.s.f として } \hat{H}_{\text{SUSY}} \text{ を作る}$$

$$V_t(x) = -\log P(x, t)$$

$$\left[V_t'(x) = -\frac{\partial}{\partial x} \log P(x, t) \right] \quad \text{①} \quad \text{SDE 登場}$$

forward $dx = f(x, t) dt + \sqrt{2D_t} dw$ $\text{①} \quad f_t, D_t$ を適当に

① $P_0(x)$ を x を生成して SDE

② $P_t(x)$ を生成 $\rightarrow P_T(x)$: 作りかた分布
 $S(x, t) = \frac{\partial}{\partial x} \log P(x, t)$ を与える?

Step up SDE の T まで

$$S(x, t) = \frac{\partial}{\partial x} \log P(x, t)$$

$$\log \hat{\psi}_t(x) + \log \psi_t(x)$$

forward $f_+(x, t) = 2\hbar \frac{\partial}{\partial x} \ell_f \tilde{\psi}_t(x) = f(x, t) \text{ になる!}$

backward $f_-(x, t) = 2\hbar \frac{\partial}{\partial x} \ell_g \psi_t(x) = -f(x, t) + 2\hbar \frac{\partial}{\partial x} \ell_g \rho$

↑
学習4.2.27を参照。